

Department of Commerce

University of Calcutta

Study Material

Cum

Lecture Notes

Only for the Students of M.Com. (Semester II)-2020

University of Calcutta

(Internal Circulation)

Dear Students,

Hope you, your parents and other family members are safe and secured. We are going through a world-wide crisis that seriously affects not only the normal life and economy but also the teaching-learning process of our University and our department is not an exception.

As the lock-down is continuing and it is not possible to reach you face to face classroom teaching. Keeping in mind the present situation, our esteemed teachers are trying their level best to reach you through providing study material cum lecture notes of different subjects. This material is not an exhaustive one though it is an indicative so that you can understand different topics of different subjects. We believe that it is not the alternative of direct teaching learning.

It is a gentle request you to circulate this material only to your friends those who are studying in Semester II (2020).

Stay safe and stay home.

Best wishes.

For

Semester-II

[Additional Materials]

Paper

CC 202 ME: Managerial Economics

Module –I

Unit - 3: Production and Cost

Unit -4: Pricing and Market Structures

(Prepared by Dr. Mahananda Kanjilal and Anindita Basu)

Perfect Competition (Mathematical Application)

- For a competitive firm, the price charged is Rs 50 and $TC = 20 + 10q + q^2$. What will be the maximum profit?

We know that the first order condition of profit maximisation is $P = MC$

$$\text{Here } MC = 10 + 2q$$

$$\therefore 10 + 2q = 50$$

$$\text{or, } 2q = 40$$

$$\therefore q = 20$$

Slope of MC at $q = 20$ will be $2 > 0$

$$\therefore \text{Profit } (\pi) = TR - TC$$

$$= Pq - TC$$

$$= 50 \times 20 - [20 + 10 \times 20 + (20)^2]$$

$$= 1000 - 620$$

$$= 380$$

2. For a competitive firm, $TC = q^3 - 14q^2 + 69q + 100$

i) What is the individual firm's supply curve?

ii) What is the industry supply curve if the industry has 100 firms?

iii) Find the amount supplied by each firm and the industry for $P = 37$.

Ans:- $TC = q^3 - 14q^2 + 69q + 100$

$$\therefore MC = \frac{d}{dq}(TC) = 3q^2 - 28q + 69$$

At equilibrium, $P = MC$.

$$P = 3q^2 - 28q + 69$$

$$\Rightarrow 3q^2 - 28q + 69 - P = 0$$

Solving for q gives $q = S_i = \frac{28 \pm \sqrt{(-28)^2 - 4 \times 3 \times (69 - P)}}{2 \times 3}$

$$= \frac{14 + \sqrt{3P - 11}}{3} \quad (\text{here we take only +ve sign})$$

Individual supply curve of industry will be the portion of MC which is ~~the~~ above the min^m point of AVC.

$$\text{Now } AVC = \frac{q^3 - 14q^2 + 69q}{q} = q^2 - 14q + 69$$

$$\text{Min } AVC = \frac{d}{dq} (AVC) \rightarrow 2q - 14$$

$$\text{At min}^m \text{ point of } AVC = \frac{d}{dq} (AVC) = 0$$

$$\Rightarrow 2q - 14 = 0$$

$$\therefore q = 7$$

We observe that $\frac{d^2(AVC)}{dq^2} = 2 > 0$, which satisfies the 2nd order condition.

$$\text{At } q = 7, \quad AVC = 7^2 - 14 \cdot 7 + 69 = 20$$

$$\therefore S_i = \frac{14 + \sqrt{3P - 11}}{3} \quad \text{if } P \geq 20$$

$$= 0 \quad \text{if } P < 20$$

ii) The industry supply curve (S) is given by

$$S = 100 S_i$$

$$= 100 \left[\frac{14 + \sqrt{3P - 11}}{3} \right] \quad \text{if } P \geq 20$$

$$\text{if } P < 20$$

$$= 0$$

$$\text{At } P = 37, \quad S_i = \frac{14 + \sqrt{3 \times (37) - 11}}{3}$$

$$= 8$$

$$\text{At } P = 37, \quad S = 100 \times S_i$$

$$= 100 \times 8 = 800$$

Monopoly (Some Mathematical Application)

1. For a monopolist $TC = 50 + 40q$ and $q = 50 - \frac{1}{2}P$.
Find out P_0 , q_0 , π_0 .

Ans: Here $q = 50 - \frac{1}{2}P$.

$$\therefore P = 100 - 2q$$

$$\therefore TR = Pq = 100q - 2q^2$$

$$\Rightarrow MR = \frac{d}{dq}(TR)$$

$$= 100 - 4q$$

$$\text{Here } TC = 50 + 40q$$

$$MC = 40$$

At equilibrium $MR = MC$.

$$\therefore 100 - 4q = 40$$

$$q_0 = 15$$

$$\therefore P_0 = 100 - 2q_0 = 70$$

$$\pi_0 = TR - TC$$

$$= P_0 q_0 - [50 + 40q_0]$$

$$= 70 \times 15 - [50 + 40 \times 15] = 400$$

2. For a discriminating monopolist, the demand functions in two markets are given as $P_1 = 50 - 2q_1$, $P_2 = 80 - 5q_2$ and $TC = 50 + 10(q_1 + q_2)$.
i) Find the price levels and also the quantities sold.
ii) Calculate elasticity of demand in each market.
iii) Calculate the total profit earned.

Ans: - The demand function in market 1 and 2 are $P_1 = 50 - 2q_1$ and $P_2 = 80 - 5q_2$

$$\therefore MR_1 = 50 - 2q_1 \text{ and } MR_2 = 80 - 5q_2$$

For a discriminating monopolist equilibrium will be set up at the point where $MR_1 = MR_2 = MC$

$$\text{Here } TC = 50 + 10(q_1 + q_2)$$

$$\therefore MC = 10.$$

Now from $MR_1 = MC$ we get $q_1 = 10$

and from $MR_2 = MC$ we get $q_2 = 7$.

$$\therefore P_1 = 50 - 2(10) = 30, \quad P_2 = 80 - 5 \times 7 = 45.$$

ii) As $MR_1 = MC$.

$$\therefore P_1 \left(1 - \frac{1}{|e_{11}|}\right) = 10.$$

$$\text{or; } 30 \left(1 - \frac{1}{|e_{11}|}\right) = 10$$

$$\therefore |e_{11}| = \frac{3}{2}$$

Again as $MR_2 = MC$

$$\therefore P_2 \left(1 - \frac{1}{|e_{21}|}\right) = 10$$

$$45 \left(1 - \frac{1}{|e_{21}|}\right) = 10$$

$$\therefore |e_{21}| = \frac{9}{7}$$

iii) Total Profit = $TR_1 + TR_2 - TC$.

$$= P_1 q_1 + P_2 q_2 - [50 + 10(q_1 + q_2)]$$

$$= 30 \times 10 + 45 \times 7 - [50 + 10(10 + 7)]$$

$$= 395.$$

3. A monopolist faces the demand curve $P = 100 - \frac{1}{2}q$ and he produces the same product in 2 plants. The cost functions for these plants are $C_1 = 10q_1$, $C_2 = 0.25q_2^2$

- i) How much will he allocate in both the markets?
- ii) How large are the profits?

Ans:- Here the producer ~~is~~ ^{is} a multi plant monopolist. So the equilibrium condition of a multi-plant monopolist is $MR = MC_1 = MC_2$

$$\text{As } P = 100 - \frac{1}{2} q$$

$$\therefore MR = 100 - q$$

$$= 100 - (q_1 + q_2), \text{ where } q = q_1 + q_2$$

$$\text{Now, } MC_1 = \frac{d}{dq_1} (TC_1) = 10$$

$$\text{and } MC_2 = \frac{d}{dq_2} (TC_2) = \frac{1}{2} q_2$$

$$\therefore MR = MC_1$$

$$100 - (q_1 + q_2) = 10$$

$$\text{or, } q_1 + q_2 = 90 \text{ — ①}$$

$$MR = MC_2$$

$$100 - (q_1 + q_2) = \frac{1}{2} q_2$$

$$q_1 + 1.5q_2 = 100 \text{ — ②}$$

Solving equation ① & ② we get $q_1 = 70$ and $q_2 = 20$.

ii) we know that $P = 100 - \frac{1}{2} q$

$$= 100 - \frac{1}{2} (q_1 + q_2)$$

$$= 55$$

$$\therefore \pi = TR - TC_1 - TC_2$$

$$= Pq - 10q_1 - 0.25q_2^2$$

$$= \cancel{50 \times 90} -$$

$$= 55 \times 90 - 10 \times 70 - 0.25 \times 20^2$$

$$= 4150$$

CC 202 ME: Managerial Economics

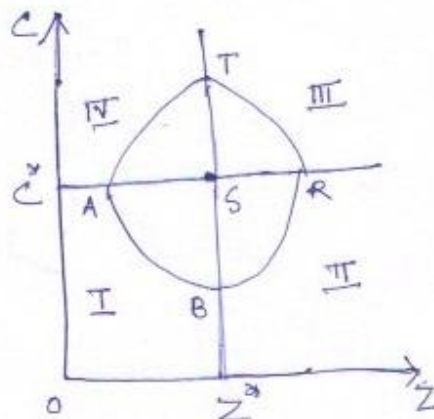
(Dr. Titas Kumar Bandopadhyay)

Some Important Aspects of the Theory of Consumer's behavior

Indifference curves, Satiations and Lexicographical Orderings:

The usual shapes of the ICs are valid only on the psychological assumptions-non-satiety, transitivity and diminishing MRS. Any or all of these conditions can be violated in reality and there is nothing necessarily pathological about such violations.

Suppose Z and C are two goods. Z^* and C^* be the maximal desired quantities of Z and C . What happens to the shape of the ICs beyond these quantities?



In the above figure, we see that rectangle OZ^*SC^* is the region of non-satiation. Any point in this region (labeled as I) represents a combination of the two goods which leaves the consumer wanting more of either or both. In this region, we see a normally shaped IC (AB). However, at any point in region II to the right of Z^* but below C^* , the consumer still wants more of C but now desires less of Z. Here, if he gets still more Z and yet remains indifferent, he must be compensated for the rise in Z by a desired rise in C. Thus the ICs in region II must have a positive slope. The reader should verify that the same argument holds for region IV in which there is too much C but Z is still desired. However, in region III, where the consumer has more than he wants of either item, the IC will again acquire its negative slope.

Thus we see that the conventional ICs lying in region I is negatively sloped and convex to the origin due to falling MRS. However, in region III though it is negatively sloped but concave to the origin due to increasing MRS. Note that at the points B and T ICs are horizontal. Similarly, at A and R ICs are vertical.

Remark: In region I both C and Z are good commodities. In region II Z is bad but C is good. Both are bad in region III. Z is good and C is bad in region IV.

Lexicographic Ordering:

The indifference curves (ICs) would be negatively sloped, i.e., they would be sloping downward towards right like the curves given in Fig. 6.2. Axiom of continuity of preferences ensures that the ICs really exist.

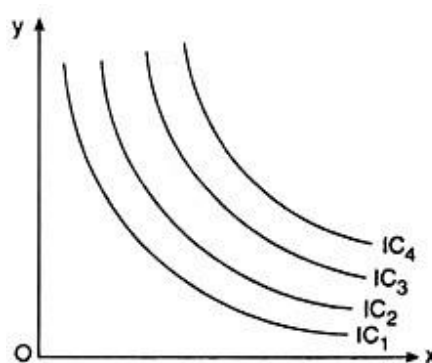


Fig. 6.2 The indifference map

However, a consumer may order his commodity combinations in such a way that would preclude the existence of ICs. Suppose that the consumer craves so much for a certain good, X, that he would prefer any combination that would have more of X, whatever may be the quantity of the other good, Y, in the bundle.

But if any two combinations contained equal amounts of X, then he would prefer the bundle with more of Y. This kind of ordering has been shown in Fig. 6.5.:

In this figure, all combinations to the right of V contain more of good X. Hence all points to the right of V are preferred to V and all points to the left of V are inferior to V. Also, for the combinations with a given amount of good X as of point V, those lying to the north of V (like U), would be preferred to V, and those lying to the south of V (like W), would be inferior to the combination V.

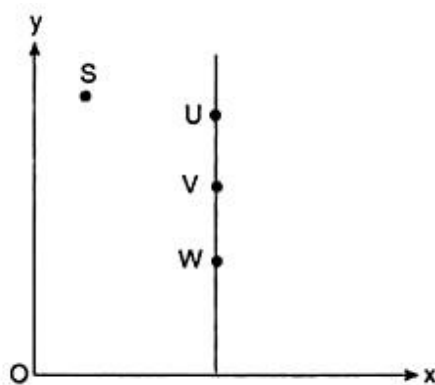


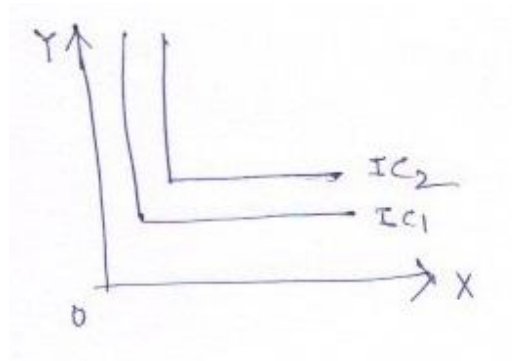
Fig. 6.5 Lexicographic ordering

Now consider another point like S. The consumer might have been indifferent between S and V by virtue of the MIB axiom. But here S would be definitely inferior to V because it lies to the left of V. In short, in the commodity space of Fig. 6.5 there are no points other than V itself which are indifferent to V.:

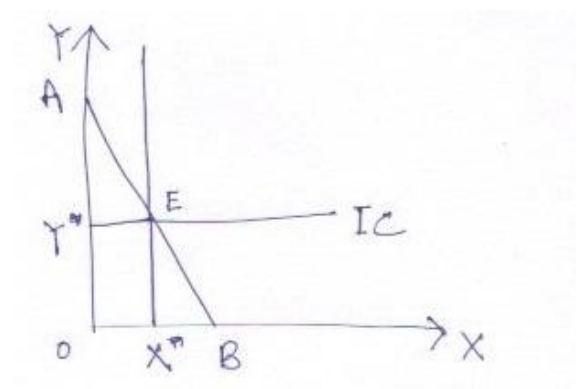
Therefore, here the indifference curve does not exist. This kind of ordering is called a lexicographic or lexical ordering. It may be noted that in the lexicographic ordering, the axiom of continuity of preferences has been violated.

Some non-standard cases of ICs;

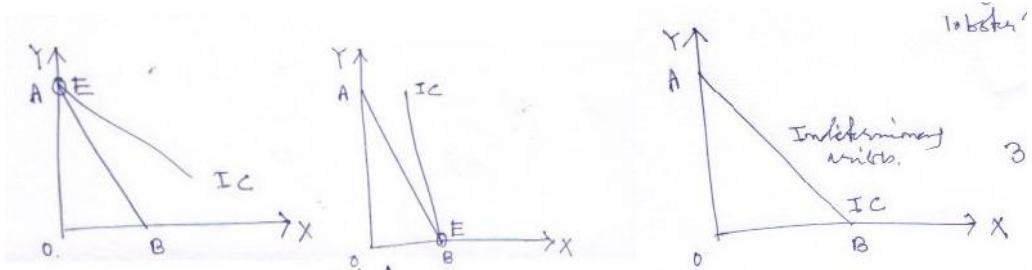
1. ICs may be L-shaped. In this case, the consumer prefers to use both goods X and Y in a strict proportion. Here, the two goods are complementary by nature.



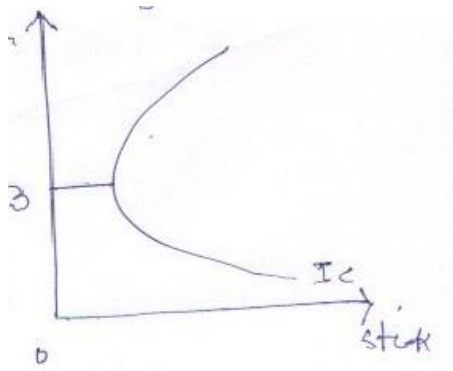
If two goods are used in fixed proportion then a rational consumer consumes both goods in constant proportion in equilibrium. This implies that equilibrium occurs at the kink points of the ICs in the commodity-commodity space.



2. If two goods are perfect substitutes then the ICs are straight lines. In this situation, a consumer may consume only one good or both goods or no such cases may arise depending on the relative positions of the IC curve and the budget line. If the two curves coincide each other then equilibrium may occur at any point on the IC and this leads to indeterminacy of equilibrium.



3. Suppose that Smith enjoys steak and lobster but believes in moderation when it comes to eating seafood. Smith thinks that steak is good but beyond 3 lobsters a day, seafood is bad. His IC for steak and lobster looks like the following figure:



Here, for 3 or fewer lobsters both steak and lobster are good commodities and lower segment of the IC has the standard properties. However, lobsters are bad for Smith when consumption of lobsters exceeds 3 per day. Beyond 3 lobsters, the IC becomes positively sloped because Smith needs more steak to compensate him for the loss of utility from the additional lobsters.

CC 202 ME: Managerial Economics

Module-II

Externality and Public Good

[Dr. Susmita Chatterjee & Dr. Nilanjana Biswas]

An item whose utilization is not determined by the individual consumer but by the public as a whole, and which is financed by taxation. A public good (or service) may be consumed without reducing the amount available for others, and cannot be withheld from those who do not pay for it. Public goods (and services) include economic statistics and other information, law enforcement, national defence, parks, and other things for the use and benefit of all. No market exists for such goods, and they are provided to everyone by Government. When the market fails to provide certain goods and services, there is a clear case for government intervention. Public goods are those goods and services provided by the government because a market failure has occurred and the market has not provided them. Sometimes it is in our benefit to not allow for a market provision. In the case of police, national defence and public education it can be argued that private provision of these services would be less desirable for a variety of reasons.

Public goods are economic products that are consumed collectively, like highways, sanitation, schools, national defence, police and fire protection. For example: • Is its use available to all E.g. Street Lighting • Does it provide for public health and safety? E.g. Police, Military • Does it provide for the general welfare of society? E.g. Education • Should the services be provided by both public and private means? E.g. Sanitation, water.

The nature of public goods: Public goods are services which must be provided collectively for two main reasons:

- Non-excludability - the goods cannot be confined to those who have paid for it
- Non-rivalry in consumption - the consumption of one individual does not reduce the availability of goods to others Examples of pure public goods include flood control systems, street lighting and national defence. A flood control system, such as the Mahanadi River- Hirakud dam Project, cannot be confined to those who have paid for the service. Also, the consumption of the service by one household will not reduce its availability to others. If left to the free market mechanism, no public goods would be provided and, as a result, there would be a clear market failure. No individual consumer would pay for a product that could be consumed for free if another household decided to purchase it.

Quasi-public goods: These are products that are essentially public in nature, but do not exhibit fully the features of non-excludability and non-rivalry. The National Highways network in India is currently available to all, but could be made excludable via a system of road tax. There is also non-rivalry in consumption, but only up to an extent. Once the road becomes congested there is rivalry in consumption. Environmental public goods: An example of an environmental public good is public

open space, which nobody would provide on their own, even though everybody benefits from it being available. Street lighting is another example of a public good.

PRIVATE GOODS A good that refrains non-payers from consuming (called excludability), and utilization of the good by one individual averts use by others (termed rival consumption). Examples include almost anything that an individual purchases at a grocery store or shopping center. The rationale for this is that private goods are privately owned and can be sold to others for a price. For efficiency, its best for these goods to be traded through markets without any direct government involvement (unless they have a market failure).

EXTERNALITY IN CONSUMPTION & PRODUCTION PROCESS Externalities are familiar in practically every area of economic activity. They are defined as third party (or spill-over) effects arising from the production and/or consumption of goods and services for which no suitable compensation is paid. Externalities can cause market failure if the price mechanism does not take into account the full social costs and social benefits of production and consumption.

CONSUMPTION EXTERNALITY This occurs when consuming a good causes either a positive or negative externality to third parties.

POSITIVE CONSUMPTION EXTERNALITY: When consuming a good gives a benefit to others. Examples include: • Going to university: Your education gives benefit to rest of society (You can teach others) • Taking medicine which prevents spread of infectious disease.

Positive externalities exist when the marginal social benefit of production and/or consumption exceeds the marginal private benefit i.e. production and/or consumption generate external benefits that may go under-valued by the market. Therefore with positive externalities the benefit to society is greater than your personal benefit. Therefore with a positive externality the Social Benefit > Private Benefit. Where, Social Benefit = private benefit + external benefit. There are ample examples of economic activities that can create positive externalities: Industrial training by firms: This can decrease the costs faced by other firms and has important effects on labour productivity. A faster growth of productivity permits more output to be produced from a given amount of resources and helps improve living standards throughout the economy. Research into new technologies which can then be distributed for use by other producers. These technology spill-over effects help to reduce the costs of other producers and cost savings might be passed onto consumers through lower prices. Education: A well educated workforce can increase effectiveness and produce other important social benefits. Progressively more policy-makers are coming to value the increased returns that might be exploited from investment in human capital.

Health provision: Improved health provision and health care reduces absenteeism and creates a better quality of life and superior living standards.

Employment creation by new small firms Flood protection system and spending on improved fire protection in schools and public arenas Arts and sporting participation and enjoyment derived from historic buildings Positive externalities and market failure Where considerable positive externalities exist, the good or service may be under consumed or under provided since the free market may fail to take into account their effects. This is because the marginal social benefits of consuming the good > private marginal benefits. In the case of external benefits from production, the marginal social cost would be < private marginal costs. Consider the example of health care. Good quality health care

brings positive spill-over effects both for the recipient of the care but also their families and associates. A well functioning health care system also reduces the scale of absenteeism from work due to sickness and illness.

NEGATIVE CONSUMPTION EXTERNALITIES: Consuming a good causes a harmful effect to third parties.

- Example: smoking causes harmful effect to those who breathe in your smoke. Consumers can create externalities when they purchase and consume goods and services.
- Pollution from cars and motorbikes
- Litter on streets and in public places
- Noise pollution from using car stereos or ghetto-blasters
- Negative externalities created by smoking and alcohol abuse
- Externalities created through the mis-treatment of animals
- Vandalism of public property
- Negative externalities arising from crime

In these circumstances the marginal social benefit of consumption will be less than the marginal private benefit of consumption. (i.e. $SMB < PMB$) This leads to the good or service being over-consumed relative to the socially most advantageous. Without government intervention the good or service will be under-priced and the negative externalities will not be taken into account. Thus there will be a loss of economic welfare.

RATIONALE FOR GOVERNMENT INTERVENTION Government Intervention in the Market In a free market economic system, scarce resources are allocated through the price mechanism where the preferences and expenditure decisions of consumers and the supply decisions of company's come collectively to determine equilibrium prices. The free market mechanism works through price signals. When demand is high, the possible profit from supplying to a market rises, leading to an expansion in supply (output) to meet rising demand from consumers. Day to day, the free market mechanism remains an immensely dominant device for determining how resources are allocated among competing ends. The government may decide to interfere in the price mechanism mostly on the justifications of wanting to alter the allocation of resources and attain what they recognize to be an improvement in economic and social welfare. All governments of every political point of view, intrude in the economy to influence the allocation of scarce resources among competing uses. Main reasons for government intervention The main reasons for policy intervention are:

- To correct for market failure.
 - To achieve a more equitable distribution of income and wealth
 - To improve the performance of the economy

Options for government intervention in markets:

There are many ways in which intervention can take place – some examples are given below: 1. Government Legislation and Regulation Parliament can pass laws that for example prohibit the sale of cigarettes to children, or ban smoking in the workplace. The laws of competition policy act against examples of price-fixing cartels or other forms of anti-competitive behavior by firms within markets. Employment laws may offer some legal protection for workers by setting maximum working hours or by providing a price-floor in the labour market through the setting of a minimum wage. The economy works with a vast and increasing amount of regulation. The government selected

regulators who can enforce price controls in a large amount of the utilities such as telecommunications, electricity, gas and rail transport. Free market economists criticize the scale of regulation in the economy arguing that it creates an unnecessary burden of costs for businesses – with a huge amount of “red tape” damaging the competitiveness of businesses. 2. Direct State Provision of Goods and Services

Because of privatization, the state-owned sector of the economy is much smaller than it was twenty years ago. State funding can also be used to provide merit goods and services and public goods directly to the population e.g. the government pays private sector firms to operate transportation buses and maintain our road network. 3. Fiscal Policy Intervention Fiscal policy can be used to modify the level of demand for different products and also the pattern of demand in an economy.

(a) Indirect taxes can be used to raise the price of de-merit goods and products with negative externalities designed to increase the opportunity cost of consumption and thereby reduce consumer demand towards a socially optimal level

(b) Subsidies to consumers will lower the price of merit goods. They are designed to boost consumption and output of products with positive externalities – remember that a subsidy causes an increase in market supply and leads to a lower equilibrium price

(c) Tax relief: The government may offer financial assistance such as tax credits for business investment in research and development. Or a reduction in corporation tax (a tax on company profits) designed to promote new capital investment and extra employment

(d) Changes to taxation and welfare payments can be used to control the overall distribution of income and wealth – for example higher direct tax rates on rich households or an increase in the value of welfare benefits for the poor to make the tax and benefit system more progressive 4. Intervention designed to close the information gap Often market failure results from consumers suffering from a lack of information about the costs and benefits of the products available in the market place. Government action can have a role in improving information to help consumers and producers value the ‘true’ cost and/or benefit of a good or service.

Examples might include: • Compulsory labelling on cigarette packages with health warnings to reduce smoking • Improved nutritional information on foods to counter the risks of growing obesity • Anti speeding television advertising to reduce road accidents and advertising campaigns to raise awareness of the risks of drink-driving • Advertising health screening programmes / information campaigns on the dangers of addiction These programmes are actually designed to change the “perceived” costs and benefits of consumption for the consumer. They don’t have any direct effect on market prices, but they seek to influence “demand” and therefore output and consumption in the long run. Of course it is difficult to identify accurately the effects of any single government information campaign, be it the campaign to raise awareness on the Aids issue or to encourage people to give up smoking. Increasingly advertisements are becoming more hard-hitting in a bid to have an effect on consumers.

THE EFFECTS OF GOVERNMENT INTERVENTION

One significant point is that the effects of different forms of government intervention in markets are never impartial – financial support given by the government to one set of producers rather than another will always generate “winners and losers”. Taxing one product more than another will

likewise have different effects on different clusters of consumers. The law of unintended consequences: Government intervention does not all the time work in the way in which it was anticipated or the way in which economic theory expects it should. Judging the effects of intervention – a useful check list: To help your evaluation of government intervention – it may be helpful to consider these questions: Efficiency of a policy: i.e. does a particular intervention direct to a better use of scarce resources among competing ends? E.g. does it improve allocative, productive and dynamic efficiency? For example - would introducing indirect taxes on high fat foods be an efficient way of reducing some of the external costs linked to the growing problem of obesity? The article below highlights the inefficiency of public policy in terms of public education

PRODUCTION EXTERNALITY Costs of production that must in due course be paid by someone other than the producer of a good or service. Production externalities are usually not deliberate and can have economic, social and environmental side effects. Production externalities can be measured in terms of the variation between the actual cost of production of the good and the real cost of this production to society at large. There are many examples of externalities, such as pollution and depletion of natural resources. A logging company can pay for the cost of a tree that they remove, but the cost of replacing an entire forest once it is gone is exponentially more than the sum of its lost trees. Expressway traffic jams and health problems that arise from breathing second hand smoke are further examples of externalities in production.

Linear Programming Problem (LPP)

The topic we have already covered:

1. Basic idea of Linear Programming Problem (LPP)
2. Formulation of LPP- (a) Maximisation and (b) Minimisation Type Problems
3. **Solution of LPP:**
 - **Graphical Method:**
 - (a) Solution of Maximisation Type LPP through Graphical Method
 - (b) Solution of Minimisation Type LPP through Graphical Method

□ Simplex Method

In class lectures I have discussed that it is not possible to obtain graphical solution to the LPP of more than two variables. On graph paper, only two variables can be accommodated. The analytic solution is also not possible because the tools of analysis are not well studied to handle inequalities. The most commonly used method for finding out the optimal solution to LPP is the simplex method which was developed by G. Dantzig in 1947.

The simplex method is a computational procedure i.e. an algorithm for solving linear programming problems. It is an iterative technique of optimisation. The simplex method consists of:

- (i) Finding a trial basic feasible solution (extreme point) to the constraint equations.
- (ii) Testing whether the initial basic feasible solution (IBFS) is optimal or not.
- (iii) Improving, if required, the first trial solution by set of rules and repeating the process until we reach an optimal solution.

➤ The Simplex Method for Maximisation Problems:

Question No. 1:

Solve the following using Simplex Method:

$$\text{Maximise } Z = 8x_1 + 16x_2$$

Subject to,

$$x_1 + x_2 \leq 200$$

$$x_2 \leq 125$$

$$3x_1 + 6x_2 \leq 900$$

$$x_1 \text{ and } x_2, \geq 0$$

Solution:

Introducing necessary slack variables, the given LPP becomes:

$$\text{Maximise } Z = 8x_1 + 16x_2 + 0S_1 + 0S_2 + 0S_3$$

Subject to,

$$x_1 + x_2 + S_1 = 200$$

$$x_2 + S_2 = 125$$

$$3x_1 + 6x_2 + S_3 = 900$$

$$x_1, x_2, S_1, S_2 \text{ and } S_3 \geq 0$$

SIMPLEX TABLEAU- I (i.e. INITIAL SIMPLEX TABLEAU)

| C _j (Contribution per unit) | | | 8 | 16 | 0 | 0 | 0 | Minimum Ratio or Replacement Ratio |
|---|---------------------|---|----------------|----------------|----------------|----------------|----------------|------------------------------------|
| Basic Variable Coefficient (C _B) | Basic Variables (B) | Basic Variable Value b (=X _B) | x ₁ | x ₂ | S ₁ | S ₂ | S ₃ | |
| 0 | S ₁ | 200 | 1 | 1 | 1 | 0 | 0 | 200/ 1 = 200 |
| 0 | S ₂ | 125 | 0 | 1 | 0 | 1 | 0 | 125/ 1 = 125 |
| 0 | S ₃ | 900 | 3 | 6 | 0 | 0 | 1 | 900/ 6 = 150 |
| Z _j | | | 0 | 0 | 0 | 0 | 0 | |
| Net Evaluation Row or Net Contribution Per Unit i.e. Δ _j = (C _j – Z _j) | | | 8 | 16 | 0 | 0 | 0 | |
| | | | | ↑ | | | | |

Since all the values of C_j – Z_j row are not either zero or negative, the above solution is not optimal. In order to obtain optimal solution we need to improve the above till all the values of C_j – Z_j row are either zero or negative (This is the rule for having optimal solution in case of a maximisation type of LPP). Since it is a maximisation type LPP, the C_j – Z_j row element having the **maximum value** shall be considered to find out key column. Here 16 in NER is the maximum value. Now each element of basic variable value is divided by corresponding element in key column in order to find out minimum ratio. The minimum ratio is the minimum value among the all elements. Here it is determined as 125. The row corresponding to 125 i.e. minimum ratio is termed as key row or pivot row. The element falling in the intersection of key row and key column is called the key/pivot element. Here it is “1”. In the next tables, we will find out the optimal solution.

(a) New Key Row Element

$$= \frac{\text{Old Key Row Element}}{\text{Key Element}}$$

(In Simplex Tableau I, the key element is 1 i.e. intersection value of Key Row and Key Column). The Sky Blue colour column is Key Column and Saffron Colour row is Key Row.

(b) Other than Key Row Element =

$$\text{Old Row Element (-) Corresponding Key Row Element} \times \frac{\text{Corresponding Key Column Value}}{\text{Key Element}}$$

$$= \text{Old Row Element (-) Corresponding Key Row Element} \times \text{Fixed Ratio}$$

SIMPLEX TABLEAU- II

| C _j (Contribution per unit) | | | 8 | 16 | 0 | 0 | 0 | Minimum Ratio or Replacement Ratio |
|---|---------------------|---|----------------|----------------|----------------|----------------|----------------|------------------------------------|
| Basic Variable Coefficient (C _B) | Basic Variables (B) | Basic Variable Value b (=X _B) | x ₁ | x ₂ | S ₁ | S ₂ | S ₃ | |
| 0 | S ₁ | 75 | 1 | 0 | 1 | -1 | 0 | 75/1 = 75 |
| 16 | x ₂ | 125 | 0 | 1 | 0 | 1 | 0 | 125/0 = - |
| 0 | S ₃ | 150 | 3* | 0 | 0 | -6 | 1 | 150/3 = 50 |
| Z _j | | | 0 | 16 | 0 | 16 | 0 | |
| Δ _j = (C _j - Z _j) | | | 8 | 0 | 0 | -16 | 0 | |
| | | | ↑ | | | | | |

Since all the values of C_j - Z_j row are neither zero nor negative, the above solution is also not an optimal solution. We need another iteration to find out optimal solution, which is shown in next table.

SIMPLEX TABLEAU- III

| C _j (Contribution per unit) | | | 8 | 16 | 0 | 0 | 0 | Minimum Ratio or Replacement Ratio |
|---|---------------------|---|----------------|----------------|----------------|----------------|----------------|------------------------------------|
| Basic Variable Coefficient (C _B) | Basic Variables (B) | Basic Variable Value b (=X _B) | x ₁ | x ₂ | S ₁ | S ₂ | S ₃ | |
| 0 | S ₁ | 25 | 0 | 0 | 1 | 1 | -1/3 | |
| 16 | x ₂ | 125 | 0 | 1 | 0 | 1 | 0 | |
| 8 | x ₁ | 50 | 1 | 0 | 0 | -2 | 1/3 | |
| Z _j | | | 8 | 16 | 0 | 0 | 8/3 | |
| Δ _j = (C _j - Z _j) | | | 0 | 0 | 0 | 0 | -8/3 | |

Since all the values of C_j - Z_j row are either zero or negative, so the above solution is optimal. Therefore, the optimal solution is x₁ = 50, x₂ = 125 and Max. Z = 8 × 50 + 16 × 125 = 2400.

➤ Artificial Variable Techniques:

In last LPP, we observed constraints with less than or equal to (i.e. ≤) type. This property together with the fact that the right hand side (R.H.S) of all the constraints is non-negative, provide us with a ready starting initial basic feasible solution (IBFS) that comprises of all slack variables.

But in many LPP, only slack variables cannot provide such a solution, where the left hand side (L.H.S) of all constraints is of either “≥” or “=” type. In such a case, we introduce non-negative artificial variables to the left hand side. The purpose of introducing artificial variables is just to obtain an initial basic feasible solution (IBFS). However, since such artificial variables have *no physical meaning* in the original model (hence the variables are called artificial variables), provisions must be made to make zero level at the optimum iteration. In other words, we use them

to start the solution and abandon them once their work has been over. There are two methods for removing artificial variables from the solution:

- (a) Big 'M' Method or Method of Penalty due to A. Charnes
- (b) The Two Phases Simplex Method due to Dantzig, Orden and Wolfe.

Here, we will restrict our discussion to only Big M Method.

Big M Method:

It has already been discussed since artificial variables do not represent any quantity relating to the decision problem, they must be driven out of the system and must not show in the final or optimal solution. This can be done by assigning an extremely high cost to them. Generally a value 'M' is assigned to each artificial variable, where M represents a number higher than any finite number. That is why the method of solving problems where artificial variables are involved are termed as the Big-M Method.

Thus, when the LPP is of minimisation type, we assign in the objective function a coefficient of + M to each of the artificial variables. On the other hands LPP with objective function of maximisation type, each artificial variable introduced has a coefficient – M.

➤ **The Simplex Method for Minimisation Problems:**

Question No. 2:

Solve the following using Simplex Method:

$$\text{Minimise } Z = 2x_1 + 8x_2$$

Subject to,

$$5x_1 + 10x_2 = 150$$

$$x_1 \leq 20$$

$$x_2 \geq 14 \text{ and } x_1 \geq 0$$

Solution:

According to the above constraints, the variable x_2 will have minimum value = 14. Therefore, let us assume that $x_2 = 14 + x_2'$. Hence the given LPP can be re-written as:

$$\text{Minimise } Z = 2x_1 + 8(14 + x_2') = 2x_1 + 8x_2' + 112$$

Subject to,

$$5x_1 + 10(14 + x_2') = 150$$

$$\text{or, } 5x_1 + 10x_2' = 10$$

$$x_1 \leq 20$$

$$x_1 \text{ and } x_2' \geq 0$$

Introducing necessary slack variable S_1 and artificial variable A_1 , the given LPP becomes:

$$\text{Minimise } Z = 2x_1 + 8x_2' + 0S_1 + MA_1 + 112$$

Subject to,

$$5x_1 + 10x_2' + A_1 = 10$$

$$x_1 + S_1 = 20$$

$$x_1, x_2', S_1 \text{ and } A_1 \geq 0$$

SIMPLEX TABLEAU- I (i.e. INITIAL SIMPLEX TABLEAU)

| C_j (Contribution per unit) | | | 2 | 8 | 0 | M | Minimum Ratio or Replacement Ratio |
|--|---------------------|---------------------------------|-------|-----------------|-------|-------|------------------------------------|
| Basic Variable Coefficient (C_B) | Basic Variables (B) | Basic Variable Value $b (=X_B)$ | x_1 | x_2' | S_1 | A_1 | |
| M | A_1 | 10 | 5 | 10 [*] | 0 | 1 | 10/10 = 1 |
| 0 | S_1 | 20 | 1 | 0 | 1 | 0 | 20/0 = - |
| Z_j | | | 5M | 10M | 0 | M | |
| Net Evaluation Row (NER) or $\Delta_j = (C_j - Z_j)$ | | | 2- 5M | 8 - 10M ↑ | 0 | 0 | |

The rule for optimisation in case of minimisation problem is that all values of NER i.e $C_j - Z_j$ are either zero or **positive**. But in the above solution, two values in NER are negative (i.e. 2- 5M and 8 - 10M). Therefore, the above solution is not an optimal solution. We need to further improve the initial basic feasible solution (IBFS) in the next iterations.

Out of these two negative values, the minimum value is 8 - 10M. The column containing 8 - 10M value is referred to as Key Column and marked by upper arrow.

SIMPLEX TABLEAU- II

| C_j (Contribution per unit) | | | 2 | 8 | 0 | M | Minimum Ratio or Replacement Ratio |
|--------------------------------------|---------------------|---------------------------------|------------------|--------|-------|-------|------------------------------------|
| Basic Variable Coefficient (C_B) | Basic Variables (B) | Basic Variable Value $b (=X_B)$ | x_1 | x_2' | S_1 | A_1 | |
| 8 | x_2' | 1 | 1/2 [*] | 1 | 0 | | 1 ÷ 1/2 = 2 |
| 0 | S_1 | 20 | 1 | 0 | 1 | | 20/1 = 20 |
| Z_j | | | 4 | 8 | 0 | | |
| $\Delta_j = (C_j - Z_j)$ | | | - 2 ↑ | 0 | 0 | | |

The above solution is also not optimal since one $C_j - Z_j$ row contains negative value. Therefore, the solution needs further improvement with the help of following table:

SIMPLEX TABLEAU- III

| | | | | | | | |
|--------------------------|-------|----|---|-----|---|--|--|
| 2 | x_1 | 2 | 1 | 2 | 0 | | |
| 0 | S_1 | 18 | 0 | - 2 | 1 | | |
| Z_j | | | 2 | 4 | 0 | | |
| $\Delta_j = (C_j - Z_j)$ | | | 0 | 4 | 0 | | |

The all the values of $C_j - Z_j$ row are either zero or positive, so the above solution is optimal. Therefore, the optimal solution is $x_1 = 2$, $x_2' = 0$ and Min. $Z = 2 \times 2 + 8 \times 0 + 112 = 116$.

❑ Variants of Simplex Method:

Some complications and variations which are very often encountered during simplex procedure are as follows:

1. Unbound Solution;
2. Multiple/ Alternative Optimum Solutions;
3. Degeneracy and Cycling;
4. Infeasible/ No Feasible Solution;
5. Unrestricted Variables.

➤ Unbound Solution (i.e. LPP with Infinite Solution):

Question No. 3:

Solve the following LPP based on Simplex Method:

$$\text{Maximise } Z = 2x_1 + x_2$$

Subject to,

$$x_1 - x_2 \leq 10$$

$$2x_1 - x_2 \leq 40$$

$$x_1 \text{ and } x_2 \geq 0$$

Solution:

Introducing necessary slack variables, the given LPP becomes:

$$\text{Maximise } Z = 2x_1 + x_2 + 0S_1 + 0S_2$$

Subject to,

$$x_1 - x_2 + S_1 = 10$$

$$2x_1 - x_2 + S_2 = 40$$

$$x_1, x_2, S_1, \text{ and } S_2 \geq 0$$

SIMPLEX TABLEAU

| C_j (Contribution per unit) | | | 2 | 1 | 0 | 0 | Minimum Ratio or Replacement Ratio |
|---|---------------------|---------------------------------|-------|-------|-------|-------|------------------------------------|
| Basic Variable Coefficient (C_B) | Basic Variables (B) | Basic Variable Value $b (=X_B)$ | x_1 | x_2 | S_1 | S_2 | |
| SIMPLEX TABLEAU- I | | | | | | | |
| Solve it (You may take help of the previous worked out sum) | | | | | | | |
| SIMPLEX TABLEAU- II | | | | | | | |
| Solve it (You may take help of the previous worked out sum) | | | | | | | |
| SIMPLEX TABLEAU- III | | | | | | | |
| 2 | x_1 | 30 | 1 | 0 | - 1 | 1 | |
| 1 | x_2 | 20 | 0 | 1 | - 2 | 1 | |

| | | | | | |
|--------------------------|---|---|-----|-----|--|
| Z_j | 2 | 1 | - 4 | 3 | |
| $\Delta_j = (C_j - Z_j)$ | 0 | 0 | 4 | - 3 | |
| | | | ↑ | | |

We can observe from the above Simplex Tableau III that S_1 column is the Key Column, but there is no positive element in that column. Hence there exists an **unbound solution** to the given LPP.

➤ **Multiple (Alternative) Solutions:**

Question No. 4:

Solve the following LPP by Simplex Method:

Maximise $Z = 2x_1 + 4x_2$

Subject to,

$2x_1 + x_2 \leq 18$

$3x_1 + 2x_2 \geq 30$

$x_1 + 2x_2 = 26$

x_1 and $x_2 \geq 0$

Solution:

Introducing necessary slack, surplus and artificial variables, the given LPP becomes:

Maximise $Z = 2x_1 + 4x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$

Subject to,

$2x_1 + x_2 + S_1 = 18$

$3x_1 + 2x_2 - S_2 + A_1 = 30$

$x_1 + 2x_2 + A_2 = 26$

x_1, x_2, S_1, S_2, A_1 and $A_2 \geq 0$

SIMPLEX TABLEAU

| C_j (Contribution per unit) | | | 2 | 4 | 0 | 0 | - M | - M | Minimum Ratio or Replacement Ratio |
|---|---------------------|---------------------------------|-------|-------|-------|-------|-------|-------|------------------------------------|
| Basic Variable Coefficient (C_B) | Basic Variables (B) | Basic Variable Value $b (=X_B)$ | x_1 | x_2 | S_1 | S_2 | A_1 | A_2 | |
| SIMPLEX TABLEAU- I | | | | | | | | | |
| Solve it (You may take help of the previous worked out sum) | | | | | | | | | |
| SIMPLEX TABLEAU- II | | | | | | | | | |
| Solve it (You may take help of the previous worked out sum) | | | | | | | | | |
| SIMPLEX TABLEAU- III | | | | | | | | | |
| 0 | S_1 | 2 | 0 | 0 | 1 | 3/4 | | | |
| 2 | x_1 | 2 | 1 | 0 | 0 | - 1/2 | | | |

| | | | | | | | | | | |
|--------------------------|-------|----|---|---|---|-----|---|---|--|--|
| 4 | x_2 | 12 | 0 | 1 | 0 | 1/4 | } | } | | |
| Z_j | | | 2 | 4 | 0 | 0 | | | | |
| $\Delta_j = (C_j - Z_j)$ | | | 0 | 0 | 0 | 0 | | | | |

Since all values in $C_j - Z_j$ row are zero, the above solution is optimal. Therefore, the optimal solution is $x_1 = 2$, $x_2 = 12$ and $\text{Max. } Z = 2 \times 2 + 4 \times 12 = 52$.

It may be mentioned here that the above solution of LPP is not a unique solution. When a non-basic variable in an optimal solution has a zero value in $C_j - Z_j$ row, then the solution is **not unique** and therefore, multiple solutions exist.

In above simple tableau III, the non-basic variable S_2 has a zero value in the Δ_j row. Thus this variable S_2 has a net zero contribution in the objective function and therefore, it can be included in the basis without changing the value of the objective function. Now if we take S_2 as the key column, the revised simplex tableau will be as follows:

| C_j (Contribution per unit) | | 2 | 4 | 0 | 0 | - M | - M | Minimum Ratio | |
|-------------------------------|-------|------------|-------|-------|-------|-------|-------|---------------|--------------------|
| (C_B) | (B) | $b (=X_B)$ | x_1 | x_2 | S_1 | S_2 | A_1 | | A_2 |
| SIMPLEX TABLEAU- III | | | | | | | | | |
| 0 | S_1 | 2 | 0 | 0 | 1 | 3/4* | } | } | $2 \div 3/4 = 8/3$ |
| 2 | x_1 | 2 | 1 | 0 | 0 | - 1/2 | | | - |
| 4 | x_2 | 12 | 0 | 1 | 0 | 1/4 | | | $12 \div 1/4 = 48$ |
| Z_j | | | 2 | 4 | 0 | 0 | | | |
| $\Delta_j = (C_j - Z_j)$ | | | 0 | 0 | 0 | 0 | | | |

SIMPLEX TABLEAU- IV

| | | | | | | | | | |
|--------------------------|-------|------|---|---|-------|---|---|---|--|
| 0 | S_2 | 8/3 | 0 | 0 | 4/3 | 1 | } | } | |
| 2 | x_1 | 10/3 | 1 | 0 | 2/3 | 0 | | | |
| 4 | x_2 | 34/3 | 0 | 1 | - 1/3 | 0 | | | |
| Z_j | | | 2 | 4 | 0 | 0 | | | |
| $\Delta_j = (C_j - Z_j)$ | | | 0 | 0 | 0 | 0 | | | |

Since all values in $C_j - Z_j$ row are also zero, the above solution is optimal. Therefore, the optimal solution is $x_1 = 10/3$, $x_2 = 34/3$ and $\text{Max. } Z = 2 \times 10/3 + 4 \times 34/3 = 52$.

➤ Degeneracy (i.e. Tie for Minimum Ratio):

Question No. 5:

Solve the following LPP by Simplex Method:

$$\text{Maximise } Z = 20x_1 + 10x_2$$

Subject to:

$$8x_1 + 10x_2 \leq 240$$

$$4x_1 + 4x_2 \leq 120$$

$$x_1, x_2 \geq 0$$

Solution:

Introducing necessary slack variables, the given LPP becomes:

$$\text{Maximise } Z = 20x_1 + 10x_2 + 0S_1 + 0S_2$$

Subject to:

$$8x_1 + 10x_2 + S_1 = 240$$

$$4x_1 + 4x_2 + S_2 = 120$$

$$x_1, x_2, S_1, S_2 \geq 0$$

Simplex Tableau: 1

| | | C_j | 20 | 10 | 0 | 0 | Minimum Ratio |
|-------------|-------|-------|-------|-------|-------|-------|---------------|
| CB_i | B | X_B | x_1 | x_2 | S_1 | S_2 | |
| 0 | S_1 | 240 | 8 | 10 | 1 | 0 | $240/8 = 30$ |
| 0 | S_2 | 120 | 4 | 4 | 0 | 1 | $120/4 = 30$ |
| Z_j | | | 0 | 0 | 0 | 0 | |
| $C_j - Z_j$ | | | 20 ↑ | 10 | 0 | 0 | |

The solution obtained in the above Simplex Table is not optimal, as all values in $C_j - Z_j$ row is neither negative nor zero. Therefore, the solution needs to be improved in subsequent iteration(s). In order to improve the initial solution, column with the maximum positive value (amongst the two columns) in $C_j - Z_j$ row i.e. 20 is taken into consideration and hence, x_1 is treated as entering variable and column pertaining to x_1 column is considered as Key Column. Let us find out Key Row in order to decide the leaving variable. The least value in the minimum ratio column relating to a particular row will be considered as key row. Here, the minimum ratio of both the rows is same i.e. 30, which lead to a problem of selection of leaving variable. This situation is called **degeneracy**. In order to resolve the tie in minimum ratio, the following steps should be taken into consideration.

Step 1: Dividing the elements of slack variables by the corresponding element in key column-

| Row | Key Column | Slack Variables | |
|-------|------------|-------------------|-------------------|
| | x_1 | S_1 | S_2 |
| S_1 | 8 | $\frac{1}{8}$ | $\frac{0}{8} = 0$ |
| S_2 | 4 | $\frac{0}{4} = 0$ | $\frac{1}{4}$ |

Step 2: Comparing the ratios in Step 1 from left to right column-wise and selection of row containing the smallest ratio-

The minimum ratio first occurs in S_2 row i.e. $0/4 = 0$. Therefore, S_2 is selected as leaving row or variable. The revised simplex tableau is shown as under:

Simplex Tableau II

| | | C_j | 20 | 10 | 0 | 0 | Minimum Ratio |
|-------------|-------|-------|-------|-------|-------|-------|---------------|
| CB_i | B | X_B | x_1 | x_2 | S_1 | S_2 | |
| 0 | S_1 | 0 | 0 | 2 | 1 | - 2 | |
| 20 | X_1 | 30 | 1 | 1 | 0 | 1/4 | |
| Z_j | | | 20 | 20 | 0 | 5 | |
| $C_j - Z_j$ | | | 0 | - 10 | 0 | - 5 | |

Since all the values of the above $C_j - Z_j$ (Simplex Tableau II) row are either zero or negative, the solution is optimal. Therefore, the optimal solution is:

$x_1 = 30, x_2 = 0$ and Maximise $Z = 20 \times 30 + 10 \times 0 = 600$

➤ **Infeasible/ No Feasible Solution:**

Question No. 6:

Solve the following LPP by Simplex Method:

Maximise $Z = - 2x_1 + x_2 + 3x_3$

Subject to,

$x_1 - 2x_2 + 3x_3 = 2$

$3x_1 + 2x_2 + 4x_3 = 1$

x_1, x_2 and $x_3 \geq 0$

Solution:

Introducing necessary artificial variables, the given LPP becomes:

Maximise $Z = - 2x_1 + x_2 + 3x_3 - MA_1 - MA_2$

Subject to,

$x_1 - 2x_2 + 3x_3 + A_1 = 2$

$3x_1 + 2x_2 + 4x_3 + A_2 = 1$

x_1, x_2, x_3, A_1 and $A_2 \geq 0$

SIMPLEX TABLEAU

| C_i (Contribution per unit) | | - 2 | 1 | 3 | - M | - M | Minimum Ratio |
|---|-------|------------|------------|-------|-------|-------|---------------|
| (C_B) | (B) | $b (=X_B)$ | x_1 | x_2 | x_3 | A_1 | |
| SIMPLEX TABLEAU- I | | | | | | | |
| Solve it (You may take help of the previous worked out sum) | | | | | | | |
| SIMPLEX TABLEAU- II | | | | | | | |
| - M | A_1 | 5/4 | - 5/4 | - 7/2 | 0 | 1 | } |
| 3 | x_3 | 3/4 | 3/4 | 1/2 | 1 | 0 | |
| Z_j | | 9/4 + 5M/4 | 3/2 + 7M/2 | 3 | - M | | |

| | | | | | | |
|--------------------------|-------------|------------|---|---|--|--|
| $\Delta_j = (C_j - Z_j)$ | -17/4 -5M/4 | -1/2- 7M/2 | 0 | 0 | | |
|--------------------------|-------------|------------|---|---|--|--|

The above solution is optimal as all values of the above $C_j - Z_j$ (Simplex Tableau II) row are either zero or negative. But in final simple tableau, A_1 is a basic variable. In presence of an artificial variable in final optimal solution, the given LPP will have no feasible solution, rather the solution is infeasible solution.

► Unrestricted/ Unconstrained Variables:

Question No. 6:

Solve the following LPP by Simplex Method:

$$\text{Maximise } Z = 2x_1 + 3x_2$$

Subject to,

$$-x_1 + 2x_2 \leq 4$$

$$x_1 + x_2 \leq 6$$

$$x_1 + 3x_2 \leq 9$$

x_1 and x_2 unrestricted in sign.

Solution:

Solution: Since x_1, x_2 are unrestricted in sign, we assume $x_1 = x_1' - x_1''$ and $x_2 = x_2' - x_2''$, so that $x_1', x_1'', x_2', x_2'' \geq 0$

The given LPP becomes:

$$\text{Max } Z = 2x_1' - 2x_1'' + 3x_2' - 3x_2''$$

$$\text{Subject to } -x_1' + x_1'' + 2x_2' - 2x_2'' \leq 4$$

$$x_1' - x_1'' + x_2' - x_2'' \leq 6$$

$$x_1' - x_1'' + 3x_2' - 3x_2'' \leq 9$$

$$x_1', x_1'', x_2', x_2'' \geq 0$$

Ans: Solving the above by Simplex Method, optimal solution is.

$$x_1 = x_1' - x_1'' = 9/2 - 0 = 9/2$$

$$x_2 = x_2' - x_2'' = 3/2 - 0 = 3/2$$

$$\text{Max } Z = 27/2$$

□ Duality:

Every LPP is associated with another unique LPP. The original (given) problem is called primal problem, while the other is called its dual problem. The solution of one problem readily provides the solution to the other one. This fact is important because situations may arise where it will be easier to solve the dual rather than to solve the primal problem. The following rules to be followed while formulating dual of primal LPP which are given below in brief:

| Primal LPP | Dual LPP |
|---|---|
| Maximisation type with \leq constraints | Minimisation type with \geq constraints |
| Minimisation type with \geq constraints | Maximisation type with \leq constraints |
| No. of variables | No. of constraints |
| No. of constraints | No. of variables |
| For “=” constraint | Unrestricted in sign |
| Unrestricted in sign | “=” constraint |

Question No. 7:

Maximise

$$Z = 100x_1 + 90x_2 + 40x_3 + 60x_4$$

Subject to

$$\begin{aligned} 6x_1 + 4x_2 + 8x_3 + 4x_4 &\leq 140 \\ 10x_1 + 10x_2 + 2x_3 + 6x_4 &\leq 120 \\ 10x_1 + 12x_2 + 6x_3 + 2x_4 &\leq 50 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

You are required to formulate the dual of the above problem. You are not required to solve by simplex method.

Solution:

Let y_1 , y_2 and y_3 be the dual variables or shadow price. Therefore, dual of the given primal problem is:

$$\text{Minimise } Z^* = 140y_1 + 120y_2 + 50y_3$$

Subject to,

$$6y_1 + 10y_2 + 10y_3 \geq 100$$

$$4y_1 + 10y_2 + 12y_3 \geq 90$$

$$8y_1 + 2y_2 + 6y_3 \geq 40$$

$$4y_1 + 6y_2 + 2y_3 \geq 60$$

$$y_1, y_2 \text{ and } y_3 \geq 0$$

(Solution by simplex method is not done).

Note:

1. Students you can easily observe that 4 variables in primal LPP becomes 3 variables in dual LPP.
2. 3 constraints in primal LPP become 4 constraints in dual.
3. Maximisation type objective function in primal becomes minimisation type objective function in dual.
4. Constraints with " \leq " type in primal become constraints with " \geq " in dual.

Question No. 8:

Find the dual of the following LPP:

$$\text{Minimise } Z = 3x_1 + 5x_2 + 10x_3$$

Subject to,

$$x_1 + 2x_2 + x_3 \geq 10$$

$$2x_1 + x_3 \geq 40$$

$$x_1, x_2 \text{ and } x_3 \geq 0$$

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 1 & 10 \\ 2 & 0 & 1 & 40 \\ 3 & 5 & 10 & \end{bmatrix} \quad \text{So, } A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 5 \\ 1 & 1 & 10 \\ 10 & 40 & \end{bmatrix}$$

Let y_1 and y_2 be the dual variables. The dual of the given primal problem is:

$$\text{Maximise } Z^* = 10y_1 + 40y_2$$

Subject to,

$$y_1 + 2y_2 \leq 3$$

$$2y_1 \leq 5$$

$$y_1 + y_2 \leq 10$$

$$y_1 \text{ and } y_2 \geq 0$$

The above questions and solutions are only explanatory and showing the type of questions. Students are also requested to practice a good number of different types of practical questions based on the above-mentioned topics from the text books already referred in class.

Customs Laws- Types of Duties

❑ Goods and Services Tax Compensation Cess [Section 3(9) of the Customs Tariff Act, 1975]:

It is levied under section 8 of the Goods and Services Tax (Compensation to State) Act, 2017. It would be applicable only on those supplies of goods and services that have been notified by the Central Government. At present, it is levied on luxury and sin goods, such as, pan masala, tobacco etc.

Any article which is imported into India shall, in addition, be liable to the goods and services tax compensation cess at such rates, as is leviable under section 8 of the Goods and Services Tax (Compensation to State) Act, 2017 on a *like article* on its supply in India, on the value of the imported article as determined u/s 3(10) or (10A) of the Customs Tariff Act, 1975.

❑ Additional Duty of Customs / Countervailing Duty (CVD) [Section 3(1) of the Customs Tariff Act, 1975]:

This duty, commonly referred to as countervailing duty (CVD), is levied on imported goods in terms of section 3 of the Customs Tariff Act, 1975 and is equal to the Central Excise duty leviable on the like goods if produced or manufactured in India.

In cases where like article is not so produced or manufactured in India, this duty will be at such rate which is leviable on the class or description of articles to which the imported article belongs. If there is more than one rate of excise duty, then the rate to be applied will be the highest. This duty is calculated on a value base of aggregate of value of the goods including landing charges and basic customs duty.

In the case of alcoholic liquors, the additional duty at present is chargeable at a uniform rate as specified by the Central Government irrespective of varying rates in force in the States.

❑ Special Additional Customs Duty / Special CVD [Section 3(5) of the Customs Tariff Act, 1975]:

It is levied to offset the effect of sales tax, VAT, local tax or other charges leviable on articles on its sale, purchase or transaction in India. It is leviable on imported goods even if article was not sold in India.

The Central Government may levy additional duty to counter balance the sales tax, value added tax, local tax or any other charges leviable in the like article on its sale, purchase or transportation in India. The rate shall be notified by the Central Government which cannot exceed 4% of the value of the imported article.

The value of the imported article shall, be the aggregate of the value determined under section 14(1) of the Customs Act, 1962 and any duty of customs chargeable on that article under section 12 of the Customs Act, 1962, and any sum chargeable on that article under any law for the time being in force as an addition to such additional duty of custom under section 3(1) and section 3(3).

Thus additional duty of custom will be levied on only few products not liable to GST.

Note: Due to introduction of GST, the applicability of additional duty of customs is very limited. GST is levied on all supplies of goods and /or services except supply of alcoholic liquor for human

consumption. Further, GST on the supply of petroleum crude, high speed diesel, motor spirit (commonly known as petrol), natural gas and aviation turbine fuel shall be levied with effect from such date as may be notified by the Government on the recommendations of the Council. Thus, applicability of additional duty of customs will be levied on the few products not leviable to GST.

❑ **Safeguard Duty [Section 8B of the Customs Tariff Act, 1975]:**

The Central Government may impose safeguard duty on specified imported goods, if it is satisfied that the goods are being imported in large quantities and they are causing serious injury to domestic industry. The safeguard duty is imposed for the purpose of protecting the interests of any domestic industry in India aiming to make it more competitive.

- **Conditions and Exemptions from Safeguard Duty-** Refer to any text book in this regard.

❑ **Countervailing Duty on Subsidised Articles [Section 9 of the Customs Tariff Act, 1975]:**

Section 9(1) provides that the countervailing duty on subsidised articles is imposed if any country directly or indirectly, pays subsidy upon the manufacture or production or exportation of any article. Such subsidy includes subsidy on transportation of such article.

- **Conditions** – Refer to any text book in this regard.

❑ **Anti-Dumping Duty [Section 9A of the Customs Tariff Act, 1975]:**

Dumping: Dumping means exporting goods to India, at prices lower than the price in the domestic market of the exporting country, subject to certain adjustments.

When the export price of a product imported into India is less than the normal value of like articles sold in the domestic market of the exporter the Central Government may, by notification in the Official Gazette, impose an anti-dumping duty not exceeding the margin of dumping in relation to such article. Anti dumping duty is country specific i.e. it is imposed on imports from a particular country.

Normal value means comparable price in the ordinary course of trade, in the exporting country, after making adjustments to the extent of conditions of sale, taxation, etc.

Computation of Anti-dumping duty: The anti dumping duty is margin of dumping or injury margin whichever is lower.

Margin of dumping = Normal Value (-) Export Price.

Normal Value means: Comparable price in the ordinary course of trade, in the exporting country, after making adjustments to the extent of conditions of sale, taxation, etc.

Injury Margin: It means difference between fair selling price of domestic industry and landed cost of imported product.

Fair Selling price: Price at which the industry have expected to charge under normal circumstances in the Indian market.

- **Provisions relating to Provisional Anti-Dumping Duty, Period of Duty, Rules relating to Anti-Dumping Duty, Assessment, Collection of Anti-Dumping Duty and No Levy u/s 9 or 9A in certain cases (Section 9B of the Customs Tariff Act, 1975)-** Refer to any text book in this regard.

Practical Questions:**Question 3:**

Mr. Raman, an importer, imported 1,000 kgs. of goods, the Cost, Insurance and Freight (CIF) value of which was US\$ 20,000. Exchange rate was US\$ 1 = Rs. 70 on the date of presentation of Bill of Entry as per CBIC. The same as per RBI on the date of presentation of Bill of Entry was US\$ 1 = Rs. 71.20. BCD was 12% and SWS @ 10%. There was no excise duty payable on these goods, if manufactured in India. As per the notification of the Govt. of India, anti-dumping duty has been imposed on such goods. The Anti-dumping duty will be equal to difference between the amount calculated @ US\$ 28.50 per kg and 'landed value' of goods.

Calculate total customs duty payable.

Solution:

| Particulars | Amount (Rs.) | Duty (Rs.) |
|--|------------------|-----------------|
| 1. CIF Value = US\$ 20,000 × Rs. 70 | 14,00,000 | - |
| Assessable Value: CIF Value | 14,00,000 | - |
| BCD @ 12% on Rs.14,00,000 (Rs. 14,00,000 × 12%) | 1,68,000 | 1,68,000 |
| SWS @ 10% on BCD (Rs. 1,68,000 × 10%) | 16,800 | 16,800 |
| Landed Value of the Imported Goods and Duty Payable | 15,84,800 | 1,84,800 |

Note: Exchange rate notified by CBIC is to be taken into consideration.

| Particulars | Amount (Rs.) |
|--|-----------------|
| 2. Rate as per Anti-dumping Notification [1,000 kgs × US\$ 28.50 × Rs. 70] | 19,95,000 |
| Total Imported Value of the Goods and Duty Payable | 1,54,900 |

3. Computation of Anti-Dumping Duty:

| Particulars | Amount (Rs.) |
|---------------------------------------|-----------------|
| Rate as per Anti-dumping Notification | 19,95,000 |
| Less: Landed Value | 15,84,800 |
| Anti-Dumping Duty Payable | 4,10,200 |

Therefore, total customs duty payable = Rs. (1,84,800 + 4,10,200) = Rs. 5,95,000.

The above question and solution is only explanatory and showing the type of questions. Students are also requested to practice a good number of different types of practical questions based on the above-mentioned topics from the text books already referred in class.

Valuation under Customs

❑ Valuation of Goods :

As per section 14(1) of the Customs Act, 1962, for the purposes of the Customs Tariff Act, 1975 or any other law for the time being in force, the value of the-

(a) imported goods and

(b) export goods

- shall be the transactional value of such goods, that is to say, the *price actually paid or payable for the goods when sold-*
- for export to India for delivery at the time and place of importation, or;
- for export from India for delivery at the time and place of exportation,
- where the buyer and seller of the goods **are not related**, and
- **price is the sole consideration for the sale**,
- subject to such conditions as may be specified in the Rules made in this behalf.

The conditions laid down above are common to imports as well as exports. Export goods are to be valued as per section 14(1) of the Customs Act, 1962. If any one of the above conditions are not satisfied, valuation for export goods should be done based on The Customs Valuation (Determination of Value of Export Goods) Rules 2007.

However, in case of imported goods, assessable value is to be determined in accordance with The Customs (Determination of Value of Imported Goods) Rules 2007.

❑ Price to be Computed with Reference to Exchange Rate:

The 'price' referred u/s 14(1) of the Customs Act, 1962 is to be calculated with reference to exchange rate as fixed by CBIC as in force on the date on which a Bill of Entry is presented u/s 46, or a Shipping Bill or Bill of Export is presented u/s 50 of the Customs Act, 1962.

In case of *Samar Timber Corporation v. ACC (1995) (Bombay)*, it was held that relevant date in respect of rate of duty payable is the date of presentation of Bill of Entry and not the date of re-presentation after correction.

❑ Transaction Value:

As per Rule 3(1), subject to Rule 12, the value of the imported goods shall be the **transaction value** adjusted in accordance with the provisions of Rule 10 of The Customs (Determination of Value of Imported Goods) Rules 2007.

According to Rule 2(1)(g), the transaction value means the value referred to in section 14(1) of the Customs Act, 1962. Section 14(1) of the Act states that transaction value means the '*the price actually paid or payable*' when sold for export to India for delivery at the time and place of importation, **where the buyer and seller of the goods are not related and price is the sole consideration for the sale**.

❑ Conditions Subject to which Transaction Value Acceptable:

As per Rule 3(2) of The Customs (Determination of Value of Imported Goods) Rules 2007, the transaction value of the imported goods shall be accepted subject to the following:

- There are no restrictions as to the disposition or use of the goods by the buyer;

- The sale or price is not subject to some conditions/ considerations for which a value cannot be determined in respect of the goods being valued;
- No part of the proceeds of any subsequent resale, disposal or use of the goods by the buyer will accrue directly or indirectly to the seller, unless an appropriate adjustment can be made in accordance with the provisions of Rule 10 of the said import valuation rules.
- The buyer and seller are not related, or where they are related, that transaction value is acceptable for customs purpose under the provisions of Rule 3(3).

❑ Rule 10 of The Customs (Determination of Value of Imported Goods) Rules 2007- Cost of Service:

The following shall be added to the invoice price (i.e. Free on Board or **FOB** value) to determine the transaction value for imported goods:

The Customs (Determination of Value of Imported Goods) Rules 2007 as amended in 2017 vide Notification No. 91/2017-CUSTOMS (NT), dated 26.09.2017 (hereinafter referred to as CVR 2007)-

| Particulars | Value in Rs. |
|---|--------------|
| Rule 10(1): | |
| Commission and Brokerage (except buying commission) | **** |
| Packing Cost (except cost of durable and returnable packing) | **** |
| Cost of Engineering, Development, Plan or Sketches (undertaken outside India) | **** |
| Royalties and License Fees | **** |
| Value of the subsequent re-sale, if payable to foreign supplier | **** |
| Value of materials supplied by the buyer free of cost | |
| Rule 10(2): | |
| Cost of Freight and Insurance upto the place of Importation- - <i>Cost of Freight if not specified shall be @ 20% of FOB*.</i> - <i>Cost of Insurance if not specified shall be @ 1.125% of FOB</i> | **** |
| Ship demurrage charges chartered vessels, lighterage or barge charges | **** |
| Cost, Insurance & Freight (CIF)/ Assessable Value (AV) | ***** |

* **Note:** If goods are imported by air, such cost of import by air shall not exceed 20% of FOB value even if the value is specified.

The following **shall not be added** to the invoice price (i.e. FOB value) to determine the transaction value for imported goods:

- Duties and taxes in India;
- Cost of erection charges in India;
- Cost of transport and insurance from port to the factory of importer in India;
- Cost of development charges in connection with imported machinery;
- Port demurrage charges and unloading charges in India;

- Any charges incurred after importation (i.e. post shipment charges, unless such charges are pre-condition for importation).

❑ Computation of Assessable Value:

Statement Showing Computation of Assessable Value for Imported Goods

| Particulars | Value in Rs. |
|---|--------------|
| Value of materials (at ex-factory price) | **** |
| Carriage/ Freight/ Insurance upto the port (sea/ air) of shipment in the exporter's country | **** |
| Charges for loading on to the ship at the shipping port in the exporter's country | **** |
| Free on Board (FOB) Value | **** |
| Add: Adjustments for Rule 10(1)- if not included above: | **** |
| Add: Adjustments for Rule 10(2)- if not included above: | **** |
| Cost, Insurance & Freight (CIF)/ Assessable Value (AV) | ***** |

❑ When Transaction Value is Rejected:

If transaction value is not accepted due to non-fulfilment of the conditions specified in Rule 3(2) of The Customs (Determination of Value of Imported Goods) Rules 2007, then the value shall be determined by proceeding sequentially through Rule 4 to 9 of CVR 2007. Those Rules are mentioned below:

- Rule 4- Transaction value of identical goods
- Rule 5- Transaction value of similar goods
- Rule 6- Determination of value where value cannot be determined under Rules 3, 4 and 5
- Rule 7- Deductive method
- Rule 8- Computed value
- Rule 9- Residual method

Students are requested to study the above mentioned Rules (in detail) from the text books already referred.

❑ Classification of Goods:

Classification enables categorisation of the goods into groups / sub-groups, in order to apply a single rate of duty on each group / sub-group. This classification is based on the concept of Harmonised System of Nomenclature (HSN). HSN is an internationally accepted coding system and the same was formulated and thereby enunciated under the General Agreement on Tariffs & Trade (GATT).

❑ Interpretation Rules under the Customs Tariff Act, 1975:

If the description read with section or Chapter notes is not enough to correctly classify the goods, then general rules of interpretation have to follow. The principles governing the appropriate classification of goods under the Tariff, as set out in the 'General Rules for Interpretation of this Schedule' to the Customs Tariff are set out below.

Rules to be applied sequentially

Classification is to be first tested on the basis of Rule 1. Only if Rule 1 does not resolve the issue, the other Rules are to be looked at sequentially.

- Rule 1- General Rule for Classification
- Rule 2- Unfinished Articles and Mixtures
- Rule 3- Classification of Goods classifiable under more than one head
- Rule 4- Akin Principle
- Rule 5- Cases/ Containers for packing goods
- Rule 6- Sub-headings

Practical Questions:

👉 Question 1:

Find out the assessable value in the case given below:

- Price charged by the exporter (FOB)- Rs. 60,00,000
- Cost of transportation to India by air- Rs. 3,00,000
- Loading and unloading charges in India- Rs. 90,000
- Cost of insurance from exporting country to India- Rs. 40,000.

Solution:

Statement showing Calculation of Assessable Value

| Particulars | Amount (Rs.) |
|---|------------------|
| Transaction value (i.e. FOB) | 60,00,000 |
| <i>Add:</i> Cost of transport, loading and unloading- if goods are imported by air, such cost shall not exceed 20% of FOB, (i.e. 20% of FOB or Rs. (3,00,000 + 90,000) = Rs. 3,90,000, whichever is less) | 3,90,000 |
| <i>Add:</i> Cost of insurance | 40,000 |
| Assessable Value/ CIF | 64,30,000 |

👉 Question 2:

Find out the assessable value in the case given below:

- Price charged by the exporter (FOB)- Situation I- Rs. 60,00,000, Situation II- Rs. 18,00,000
- Cost of transportation to India by air- Rs. 3,00,000
- Loading and unloading charges in India- Rs. 90,000
- Cost of insurance from exporting country to India- Not available.

Solution:

Statement showing Calculation of Assessable Value

| Particulars | Amount (Rs.) | Amount (Rs.) |
|------------------------------|--------------|--------------|
| | Situation I | Situation II |
| Transaction value (i.e. FOB) | 60,00,000 | 18,00,000 |

| | | |
|--|------------------|------------------|
| <i>Add:</i> Cost of transport, loading and unloading- if goods are imported by air, such cost shall not exceed 20% of FOB (i.e. 20% of FOB or Rs. 3,90,000 (actual cost), whichever is less)- it is not applicable when the goods are transported by any other mode | 3,90,000 | 3,60,000 |
| <i>Add:</i> Cost of insurance (when cost is not ascertainable, it will be @ 1.125% of FOB (i.e. Rs.60,00,000 × 1.125%) and (i.e. Rs.18,00,000 × 1.125%) | 67,500 | 20,250 |
| Assessable Value/ CIF | 64,57,500 | 21,80,250 |

 **Question 3:**

Make in India Ltd. imported a machine at an invoice value of £ (GBP) 10,000. The sum includes £ 2,000 attributable to post importation activities to be carried out by the seller (post shipment expenditure was not a pre-condition for such import). The company had supplied raw materials worth £ 500 to the seller for manufacture of the said machine. The importer exported these goods by vessel and actual cost of transport was £ 1,500 and lighterage and barge charges in India were Rs. 50,000. Ship demurrage charges were Rs. 10,000. The importer also incurred in India Rs. 25,000 for transportation of goods from port of entry to Inland Container Depot (ICD). Insurance charges were not known. Rate of exchange notified by CBIC: £ 1 = Rs. 88.

You are requested to determine the assessable value of the machine.

Solution:

Statement showing Calculation of Assessable Value

| Particulars | Amount (£) |
|---|---------------------|
| Invoice Price | 10,000 |
| <i>Less:</i> Cost of post-shipment expenditure (Note 1) | 2,000 |
| | 8,000 |
| <i>Add:</i> Cost of raw materials supplied by the importer (Note 2) | 500 |
| FOB Value as per Customs | 8,500 |
| | Amount (Rs.) |
| FOB (£ 8,500 × Rs. 88) | 7,48,000 |
| <i>Add:</i> Ship demurrage charges | 10,000 |
| Lighterage and barge charges | 50,000 |
| <i>Add:</i> Freight charges (£ 1,500 × Rs. 88) | 1,32,000 |
| <i>Add:</i> Cost of insurance (Rs. 7,48,000 × 1.125%) | 8,415 |
| Assessable Value/ CIF | 9,48,415 |

Note:

1. Since post shipment expenditure is not a pre-condition for importation, such expenditure is deducted from the value of the machine.

2. Cost of raw materials supplied by the importer has been added while calculating assessable value on the assumption that it was supplied at free of cost.
3. Cost of transport of the imported goods includes ship demurrage and lighterage charges [Explanation to Rule 10(2) of The Customs (Determination of Value of Imported Goods) Rules 2007].
4. Since insurance charges are not known, it is included in assessable value @ 1.125% of FOB value of the machine [Third Proviso to Rule 10(2) of The Customs (Determination of Value of Imported Goods) Rules 2007].

The above questions and solutions are only explanatory and showing the type of questions. Students are also requested to practice a good number of different types of practical questions based on the above-mentioned topics from the text books already referred in class.

Duty Drawback

❑ Duty Drawback (Sections 74- 76 of Customs Act, 1962):

Under the duty drawback scheme, if import duty paid goods are exported with or without any value addition, the import duties and other taxes paid on such goods at input level are refunded in the form of duty drawback. Duty drawback is basically a refund of import duties. There are two variants of duty drawback scheme under Customs.

1. Re-exportation of duty paid imported goods [Section 74]
2. Export of final products/ processed goods using duty paid imported material [Section 75]

In both the cases, there are three common features.

- (a) There is import of some goods;
- (b) The imported goods suffered import duty;
- (c) The same goods in same form or in a different form have been exported.

❑ Drawback Allowable on Re-Export of Duty Paid Goods (Sections 74 of Customs Act, 1962):

The elements necessary to consider a claim for Drawback u/s 74 of Customs Act, 1962 are as follows:

- The goods on which the drawback is claimed must have been previously imported;
- Import duty must have been paid on these goods when they were imported;
- The goods must be entered for re-export within two years from the date of payment of duty. However, it is provided that in any particular case this period of two years may, on sufficient cause being shown, be extended by the Board by such further period it may deem fit;
- The goods are identified to the satisfaction of the Assistant Commissioner of Customs as the goods that were imported;
- The goods must be actually re-exported to any place out-side India;
- The goods must be capable of easy identification; and
- The market price of such goods must not be less than the amount of drawback claimed.

❑ Rates of Drawback of Import Duty Admissible Under Section 74 of Customs Act, 1962:

Study relevant portion of the book already referred in class.

❑ Drawback on Imported Materials Used in the Manufacture, Processing or Operation of Goods Which are Exported (Section 75 of Customs Act, 1962):

Study relevant portion of the book already referred in class.

❑ Payment of Interest on Drawback (Section 75A of Customs Act, 1962):

Study relevant portion of the book already referred in class.

❑ Prohibition and Regulation of Drawback in Certain Cases (Section 76 of Customs Act, 1962):

Independent of other conditions which are laid down in Sections 74 and 75 of Customs Act, 1962 and in the Drawback Rules, no drawback will be granted:

- (a) in respect of any goods the market price of which is less than the amount of drawback due thereon; and
- (b) where drawback due on any goods is less than Rs. 50.

Also, if the Central Government is of opinion that specified goods on which drawback is claimed are likely to be smuggled back into India, it may stipulate that drawback be paid subject to certain conditions.

Other Prohibitions and Regulations on Drawback are as under:

- (c) CENVAT credit claim is on inputs and input services then no duty drawback is allowed. However, if the goods have already suffered the customs duty then duty drawback is allowed to the extent of customs duties.
- (d) Duty drawback is not allowed if the exporter has already availed the Duty Entitlement Pass Book (DEPB) or other export incentives.
- (e) If the sale proceeds not received within the time period allowed by Reserve Bank of India.
- (f) Export to Nepal and Bhutan and the export proceeds are not received in hard currency (it means USD, GBP or Pounds).
- (g) drawback in respect of iron and steel, cement and rice is not allowed. [w.e.f. 29-5-2008]
- (h) duty drawback is more than 1/3rd of market value of exported goods, then amount of duty drawback is restricted to 1/3rd of market value.
- (i) No amount or rate of drawback is to be determined except where the amount of drawback exceeds or equal to Rs. 500/- or it is 1% or more of the FOB value of export

Practical Question:**👉 Question 1:**

Calculate the amount of duty drawback allowable u/s 74 of the Customs Act, 1962 in the following cases:

- (i) Mr. X imported wearing apparel and paid Rs. 35,000 as import duty. As he did not like the same, these were re-exported after 20 days.
- (ii) Softonic Ltd. imported 15 high value computer systems paying total import duty of Rs. 40 lakhs. Immediate after receipt, some technical problems were observed and the computer systems were returned to foreign exporter after 2 months without using them at all.

Solution:

- (i) In case of Mr. X the amount of duty drawback will be nil, as no duty drawback available on wearing apparel.
- (ii) Softonic Ltd. will get 98% of import duty paid as duty drawback. So, the amount of duty drawback = 98% × Rs. 40 Lakhs = Rs. 39.20 Lakhs.

The above question and solution is only explanatory and showing the type of questions. Students are also requested to practice a good number of different types of practical questions based on the above-mentioned topics from the text books already referred in class.

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CC 204: Indirect Tax and Corporate Tax Planning (ITCTP)

(Prof. S. Basu & Dr. A. Kundu)

5. Taxation of Companies

Tax payable by a company is subject to Minimum Alternative Tax (MAT) u/s 115JB. In other words, a company is liable to pay tax which is higher of:

- (a) Tax @ 30% plus surcharge (if any) plus HEC @ 4% on taxable income computed under normal provisions of the Income Tax Act (ignoring section 115JB).
- (b) Tax @ 18.5% plus surcharge (if any) plus HEC @ 4% on book profit computed under section 115JB.

However, the companies can now pay tax at a lower rate. Such provisions are discussed below:

A. For existing companies:

1) Section 115BA (w.e.f. A.Y. 2017-18): A domestic company (not opting section 115BBA, 115BAB) can pay tax @ 25% subject to certain conditions. However, companies opting section 115BA are subject to MAT under section 115JB.

2) Section 115BAA (w.e.f. A.Y. 2020-21): A domestic company (not opting section 115BA) can pay tax @ 22% subject to certain conditions. The companies opting section 115BAA are not subject to MAT under section 115JB.

B. For newly established companies

3) Section 115BAB (w.e.f. A.Y. 2020-21): A newly established manufacturing company (set up and registered on or after 01.10.2019 and commenced manufacturing on or before 31.03.2023) can pay tax @ 15% subject to certain conditions. The companies opting section 115BAB are not subject to MAT under section 115JB.

Conditions (applicable for all of the above cases):

- i) Such companies cannot enjoy the benefits of section 10AA, additional depreciation, section 32AD, section 33AB, section 33ABA, any outside payment in respect of section 35 (revenue and capital expenditure for in-house scientific research are allowable), section 35AD, section 35CCC, section 35CCD, deductions under chapter VIA (80-IA to 80Q except 80JJAA). However, in case of section 115BA benefits of deduction u/s 32AC and 35AC are not available.
- ii) Any brought forward loss attributable to the abovementioned sections cannot be set off.
- iii) Once option is given, it cannot be subsequently withdrawn.

So, from tax planning purpose, a corporate assessee may consider the following options:

- a) Whether to pay tax as per normal provisions or opt for section 115BA or 115BAA.
 b) Whether to pay tax as per normal provisions or opt for section 115BAB.

6. Financial Management Decisions

A. CAPITAL STRUCTURE

Capital structure of a company or a project financing may comprise – equity shares, preference shares, debentures, long-term loans, etc.

In order to ascertain acceptable capital structure/ project financing one has to consider different options. The option which gives highest EPS or return on equity share capital can be chosen.

| <u>Computation of EPS/ Return on equity share capital</u> | | Rs. |
|---|--|----------|
| EBIT (which will be same for all options) | | **** |
| Less: Interest | | **** |
| | EBT | **** |
| Less: Tax | | **** |
| | EAT | **** |
| Less: Preference dividend | | **** |
| DDT on such dividend | | **** |
| | Earnings attributable to equity shareholders | **** (X) |

$$\text{EPS} = (X / \text{No. of equity shares})$$

$$\text{Return on equity share capital (ROE)} = (X / \text{Equity share capital}) \times 100$$

Decision will be same in both the cases.

B. DIVIDEND POLICY

Dividend declared by company is subject to tax implications both in the hands of declaring company and the shareholders. Since, we are discussing about corporate tax planning, the provisions are discussed after considering that shareholders are also companies. In other words, it is a case of inter-corporate dividend.

(I) Taxability in the hands of shareholders:

(a) Dividend received from a domestic company is fully exempted u/s 10(34). Any expense for earning dividend is not deductible.

(b) Dividend received from a foreign company is fully taxable subject to section 115BBD. Expenses for earning dividend are deductible except in case of section 115BBD.

(c) Section 115BBD: Tax on certain dividends received from foreign company: An Indian company received dividend [except u/s 2(22)(e)] from a specified foreign company has to pay tax @ 15% plus surcharge (if any) plus HEC @ 4%.

Specified foreign company means a foreign company in which the Indian company holds 26% or more of equity share capital.

(d) Deemed dividend u/s 2(22)(a) – (e) are not taxable.

(II) Taxability in the hands of declaring company

(a) A domestic company who declared dividend on both equity and preference shares (whether interim or final) has to pay Dividend Distribution Tax (DDT) u/s 115-O.

(b) A domestic company has to pay DDT @ 20.55529% on deemed dividend u/s 2(22)(a) – (d). However, rate of DDT in case of sec 2(22)(e) is 34.944%.

(c) Section 115-O: Tax on distributed profits of domestic companies: A domestic company is liable to pay DDT @ 15% (after grossing up) plus surcharge @ 12% plus HEC @ 4% (i.e. 20.55529%) on dividend and deemed dividend [except section 2(22)(e)] subject to relief.

(d) Relief in case of inter-corporate dividend: When a domestic holding company received dividend from its subsidiary company within the same financial year in which dividend is declared by the holding company, the dividend received from subsidiary company to be deducted from dividend declared for computation of tax payable u/s 115-O subject to the following conditions:

(i) If subsidiary company is a domestic company and paid DDT u/s 115-O.

(ii) If subsidiary company is a foreign company and dividend received is taxable in the hands of holding company u/s 115BBD.

C. BONUS SHARES

(I) In the hands of declaring company:

(i) Bonus to equity shareholders – No taxability.

(ii) Bonus to preference shareholder – Deemed dividend u/s 2(22)(b) and the company has to pay DDT.

(II) In the hands of shareholders:

No tax implication on receipt of bonus shares. However, subsequent sale of bonus shares will attract capital gains.

D. BUYBACK OF SHARES (WHETHER LISTED OR UNLISTED)

(I) In the hands of company:

Under section 115QA, a company has to pay additional income tax on distributed income on buyback of unlisted or listed (w.e.f. 05.07.2019) shares @ 20% plus surcharge @ 12% plus HEC @ 4% i.e. 23.296%. Distributed income means 'Consideration paid by company on buyback' minus 'Consideration received by company on issue of shares'. Such tax has to be remitted within 14 days from the date of payment of consideration. Otherwise, the company is liable to pay interest u/s 115QB @ 1% p.m. or part of month.

(II) In the hands of shareholders:

Fully exempted in the hands of shareholders u/s 10(34A).

Problem 1 (M.Com CU 2019):

GreyMind Ltd., a manufacturing company wishes to expand its business which requires an outlay of Rs.30,00,000. The company is considering the following alternatives:

| | Alternative:1 | Alternative:2 | Alternative:3 |
|-------------------------------------|---------------|---------------|---------------|
| Equity Shares of Rs.10 each | 30,00,000 | 20,00,000 | 10,00,000 |
| 9% Preference Shares of Rs.100 each | - | - | 5,00,000 |
| 12% Debentures of Rs.100 each | - | 10,00,000 | 5,00,000 |
| 15% Long-term loan from SBI | - | - | 10,00,000 |

Rate of corporate tax is 31.2%; rate of DDT is 20.55529% and rate of return is 15% (before tax).

You as a tax consultant are required to advise the company about the best option.

Solution:

| Particulars | Alternative 1 | Alternative 2 | Alternative 3 |
|---|---------------|---------------|---------------|
| | Rs | Rs. | Rs. |
| EBIT (15% on Rs. 30,00,000) | 4,50,000 | 4,50,000 | 4,50,000 |
| Less: Interest on Debentures | - | 1,20,000 | 60,000 |
| Less: Interest on Bank Loans | - | - | 1,50,000 |
| EBT | 4,50,000 | 3,30,000 | 2,40,000 |
| Less: Tax @ 31.2% | 1,40,400 | 1,02,960 | 74,880 |
| EAT | 3,09,600 | 2,27,040 | 1,65,120 |
| Less: Preference Dividend | - | - | 45,000 |
| Less: DDT on Preference Dividend | - | - | 9,250 |
| Earnings attributable to equity shareholders (A) | 3,09,600 | 2,27,040 | 1,10,870 |
| No. of Equity Shares (B) | 3,00,000 | 2,00,000 | 1,00,000 |
| EPS [A/B] | 1.032 | 1.135 | 1.109 |
| ROE [A/Equity Share Capital] × 100 | 10.32% | 11.35% | 11.09% |

Therefore, GreyMind Ltd. should adopt Alternative 2 because of highest EPS or ROE.

Problem 2:

MindTree Ltd. is considering to choose out of the following two alternatives:

(I) Pay dividend of Rs.150 lakhs; or

(II) Pay dividend of Rs.100 lakhs and bonus shares amounting Rs.50 lakhs to its equity shareholders during February 2020.

You are required to advise the company on the above alternatives assuming that lower payment of dividend will not affect the market price of its equity shares. For this, the company has also provided the following details of dividend received by it during the previous year 2019-20:

| Dividend received from | Nature of the Company | Total Amount of dividend declared (Rs.) | Date of declaration | Date of payment | Holdings in the nominal value of the equity capital by MindTree Ltd. |
|------------------------|-----------------------|---|---------------------|-----------------|--|
| GP Ltd. | Domestic | 89,90,000 | 21/09/2019 | 30/09/2019 | 70% |
| Snow White Inc. | Foreign | 73,50,000 | 05/08/2019 | 05/09/2019 | 61% |

GP Ltd. has paid DDT u/s 115-O on its dividend distribution amount during the previous year 2019-20. Applicable Dividend Distribution Tax rate is 20.55529%.

Solution:

Computation of DDT u/s 115-O payable by MindTree Ltd. [P.Y. 2019-20; A.Y. 2020-21]

| Particulars | Alternative 1 | | Alternative 2 | |
|---|------------------|-------------------|------------------|-------------------|
| | Rs. | Rs. | Rs. | Rs. |
| Dividend distributed by MindTree Ltd. | | 150,00,000 | | 100,00,000 |
| Less: Dividend received from domestic subsidiary GP Ltd. during P.Y. 2019-20 (DDT paid) [Rs.89,90,000 × 70%] | 62,93,000 | | 62,93,000 | |
| Dividend received from foreign subsidiary Snow White Inc. during P.Y. 2019-20 (Tax would be paid u/s 115BBD by MindTree Ltd.) [Rs.73,50,000 × 61%] | <u>44,83,500</u> | | <u>44,83,500</u> | |
| | | <u>107,76,500</u> | | <u>107,76,500</u> |
| Net distributed profit for DDT | | 42,23,500 | | nil |

Alternative I: DDT payable on dividend u/s 115-O on Rs.42,23,500 @ 20.55529% is Rs.8,68,153.

Alternative II: No DDT on dividend as net distributed profit is nil after relief. Bonus to equity shareholders is not taxable in the hands of the declaring company. Hence, no DDT is payable.

So, MindTree Ltd. should opt for alternative II.