Department of Commerce

University of Calcutta

Study Material

Cum

Lecture Notes

Only for the Students of M.Com. (Semester II)-2020

University of Calcutta

(Internal Circulation)

Dear Students,

Hope you, your parents and other family members are safe and secured. We are going through a world-wide crisis that seriously affects not only the normal life and economy but also the teaching-learning process of our University and our department is not an exception.

As the lock-down is continuing and it is not possible to reach you face to face class room teaching. Keeping in mind the present situation, our esteemed teachers are trying their level best to reach you through providing study material cum lecture notes of different subjects. This material is not an exhaustive one though it is an indicative so that you can understand different topics of different subjects. We believe that it is not the alternative of direct teaching learning.

It is a gentle request you to circulate this material only to your friends those who are studying in Semester II (2020).

Stay safe and stay home.

Best wishes.

For

Semester-II

[Additional Materials]

SERIES -II

Paper CC202: Managerial Economics (Module II)

(Dr.Samarpita Seth)

Chapter 7: Factor Pricing Under different Market Forms

In this chapter we will discuss the determination of price for different factors of production. Traditionally there are four factors of production viz., land, labour, capital and organization. Remunerations for these factors are called rent, wage, interest and profit respectively. Prices of such factors are determined through interaction of factor demand and factor supply (like product price determination). However, shape of factor demand (and factor supply) depends on the nature of product market competition . Factor demand curve under competitive product market will be different from that under monopolistic product market. Because of this feature, factor demand is also called **derived demand.**

Since labour is the most important input widely used in different production, we will restrict our discussion of factor price determination to labour market only. Price determination of other factors will follow same mechanism. We will carry out our discuss price determination of labour under following three different market structure:

I. Competitive Factor Market
II. Factor Market with Monopsony power
III. Factor Market with Monopoly power

I. Competitive Factor Market

When factor market (here labour) is competitive price of labour i,e, wage is fixed for individual seller and the corresponding labour supply curve faced by a firm will be a straight line parallel to horizontal axis (figure 1)

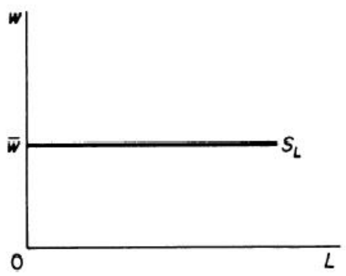

Figure1 (Source: A. Koutsoyiannis, Modern Microeconomics)

Given the supply curve, our objective is to find out the demand curve for labour in the competitive factor market.

I a. Demand curve for labour under single variable input

We assume that firm is producing output using two inputs labour and capital. Also it is assumed that labour is the variable input and capital is the fixed input for production. Since labour demand curve is derived demand, it will ultimately depend on firm's decision to produce output. Here we introduce the concept of marginal revenue product of labour (MRP_L). It is given by $MRP_L = MP_L \times MR$ where MP_L is the marginal product of labour showing additional output generated by additional employment of labour, MR is the marginal revenue showing additional revenue generated by additional output. Thus MRP_L is the additional labour generating additional revenue in the product market. Naturally a firm will purchase/ employ labour as long as marginal revenue product of labour is equated to marginal cost of employment of labour, Under competitive factor market, fixed wage rate **w** is the marginal cost of employment of labour. So the equilibrium in the factor market is determined by the equation:

$$MRP_L = w \qquad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

This labour demand curve and labour market equilibrium are shown in figure 2a and figure 2b respectively.

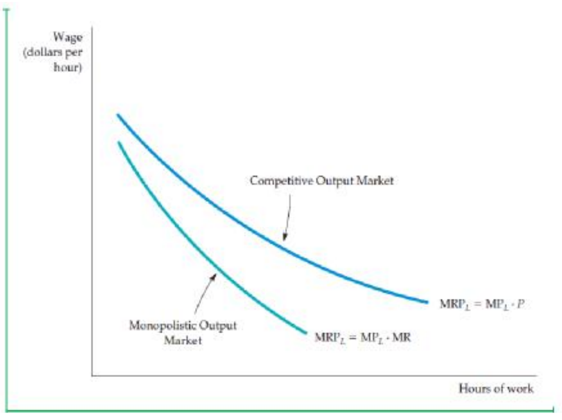

Figure2a

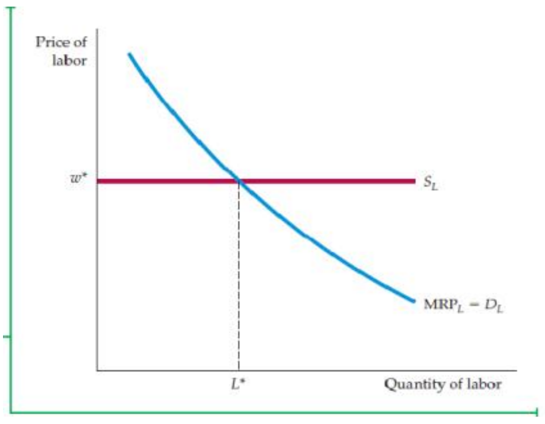

Figure 2b

(Source : Pindyck and Rubinfeld, Microeconomics)

From figure 2a it is clear that when output market is perfectly competitive, $MRP_L = MP_L.MR = MP_L.P = VMP_L$ where VMP_L stands for value of marginal product of labour. This happens because under perfectly competitive output market, $P=MR$. Hence MRP_L and VMP_L will coincide. However, if output market is monopolistic, $P > MR$. Hence VMP_L curve will lie above MRP_L curve. Figure 2b shows that equilibrium in labour market is determined by intersection of MRP_L curve and supply curve of labour. w^* is the equilibrium wage and L^* is the equibrium employment of labour.

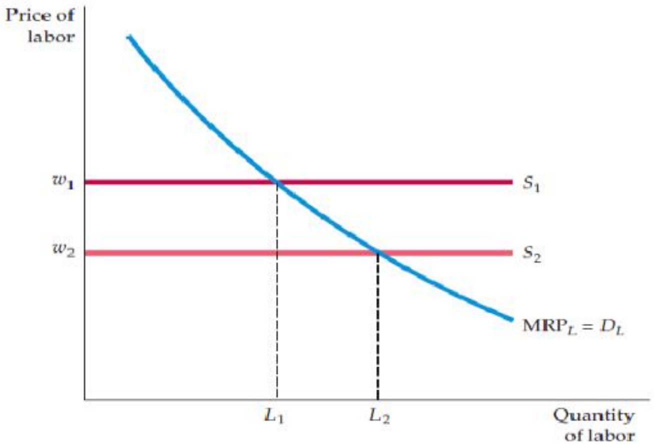

Figure3 (Source : Pindyck and Rubinfeld, Microeconomics)

Figure 3 shows the labour demand corresponding to various wage rate. When wage rate is w_1 labour demand is L_1 Similarly when wage rate is w_2, corresponding labour demand is L_2. Joining the locus of various (w, L) we obtain the labour demand curve which is nothing but our MRP_L curve. So it can be concluded that under competitive factor market (with single variable input, labour) MRP_L curve is the labour demand curve.

I b. Demand curve for labour under several variable input

Here we assume that both labour and capital are variable input. So if wage rate falls, initially firm will increase employment of labour because of its increasing productivity. Increase in MP_L will cause increase in MRP_L and firm will hire more labour to make $MRP_L=w$. However, there is an additional effect. Since capital is also variable here, a decrease in wage allows producer of capital goods to increase their production through hiring of more labour. Hence marginal product of capital also increases which in turn, encourages firm to rent more capital as well as hire more labour. Marginal product of labour increases further because of use of more capital. Hence Marginal Revenue Product (MRP_L) curve will shift to the right causing additional labour employment. This is shown in figure 4.

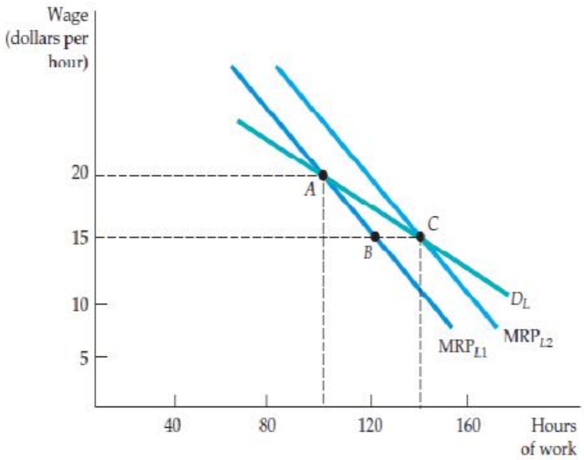

Figure 4 (Source : Pindyck and Rubinfeld, Microeconomics)

In figure 4, A to B along MRP_{L1} is the primary effect of labour employment due to decrease in wage. But B to C is the secondary effect on labour employment caused by shift of MRP_{L1} to MRP_{L2}. Joining A and C we get labour demand curve under competitive factor market when several inputs are variable. Comapring figure 3 with figure 4 it can be mentioned that labour demand will be relatively elastic when several inputs are variable.

Industry Demand Curve for labour

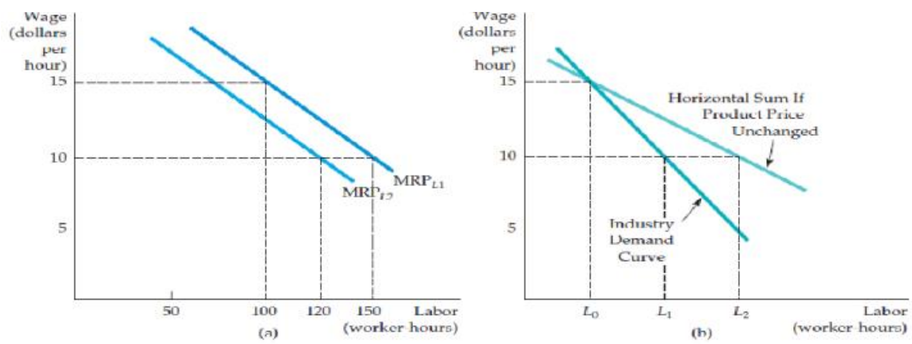

Figure5: (Source : Pindyck and Rubinfeld, Microeconomics)

In figure 5, industry labour demand curve is derived from firm's labour demand curve. Figure 5a shows MRP_{L1} is the firm labour demand curve assuming product price is fixed. Hence industry demand curve for labour will be horizontal sum of firm labour demand curve (figure 5b). However, But when wage rate falls (from15 dollar to 10 dollar), price of the product also falls (due to shortage of demand in the product market). Hence labour demand shifts leftward from MRP_{L1} *to* MRP_{L2} reducing labour employment. Figure 5b shows Industry demand curve with variable product price is much more inelastic compared to industry demand curve under fixed product price.

Now we come to market supply curve of labour. Already we know from figure 1 that individual supply curve of labour will be a horizontal straight line, indicating that firm can hire as many labour as they wish with a fixed wage rate. However, market supply curve for labour will be first upward sloping then backward bending as shown by figure 6.

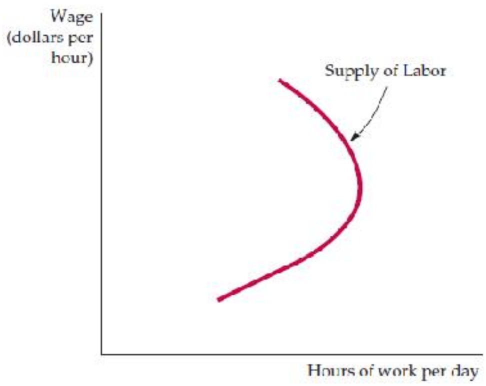

Figure 6 (Source : Pindyck and Rubinfeld, Microeconomics)

Reason for such shape of labour supply is as follows. Each day is divided into two period: work time and leisure. Increasing work time implies decrease in leisure and vice vera. As wage rate increases, intially workers engage in more work by forgoing leisure. So labour supply is upward sloping her. However, after certain period of work , as wage rate increases further, their work time will be less and it will be substituted by more leisure compared to earlier period. Hence labour supply will be backward bending.

Equilibrium in a competitive Factor Market

Equilibrium wage in the factor market in determined by the intersection of market demand for labour and market supply of labour. Figure7 shows such equilibrium under two alternative product market conditions.

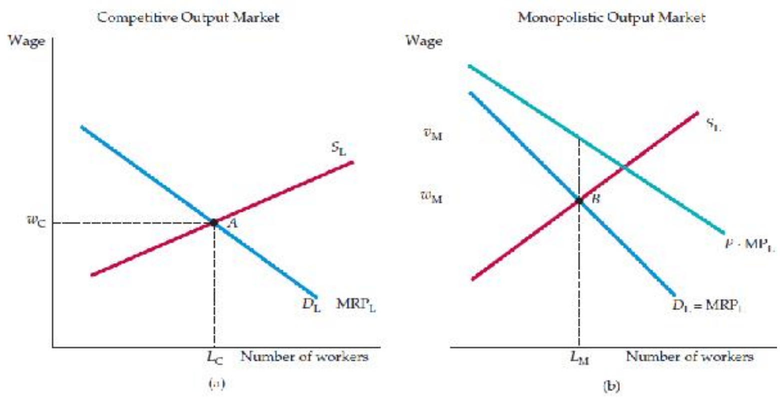

Figure 7 (Source : Pindyck and Rubinfeld, Microeconomics)

Figure 7a shows competitive factor market equibrium when firm faces perfect competition in output market. Labour is getting wage according to MRP_L (=VMP_L) as discussed earlier. However, under monopolistic output market (figure 7b) labour will be paid according to MRP_L which is less than their marginal product (VMP_L). Comparing Figure 7a and 7b, v_M=w_c=VMP_L. But, w_M = MRP_L and w_M< v_M (since P>MR). The difference between w_M and v_M is called monopolistic exploitation of labour. Because this wage differential is generated fundamentally because of monopolistic product market.

II. Factor Market with Monopsony power

Now we introduce a new concept viz, firm is a monoposonist (single buyer) in the factor market. As a single buyer firm will exercise its monoposonistic power to determine wage rate. Monopsonist buyer will reach equibrium where marginal value (*MV*) is equal to marginal expense (*ME*). Here *MV* indicates the additional value that is being generated from an additional employment of labour. In the present context, $MV = MRP_L$. Again, *ME* is the marginal expense (additional cost) for hiring additional labour. It is generated from average expense (*AE*), per unit expenditure for hiring labour. *AE* is actually the wage rate determined from labour supply curve. This is shown in figure 8.

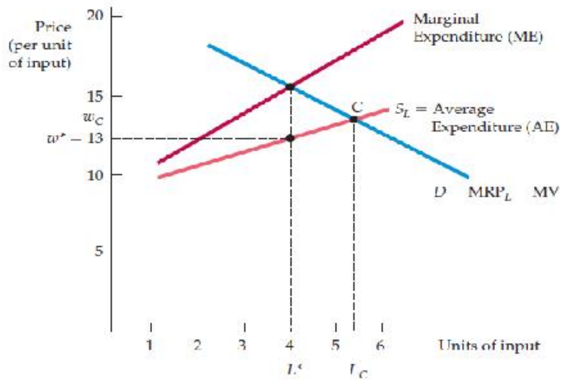

Figure 8 (Source : Pindyck and Rubinfeld, Microeconomics)

It should be pointed out that when buyers are competitive AE curve (labour supply curve of the firm) is a horizontal straight line indicating AE=ME = w_c where w_c is the competitive wage earned the labour. However, figure 8 shows when firm has monopsonist power, supply curve of labour (*AE*) is upward sloping (since firm and market labour supply are equivalent) and *ME* will lie above *AE*. Monopsonist will pay w^* which is lower than competitive wage w_c and employ L^*, lower than competitive employment L_c.

III. Factor Market with Monopoly power

Just like buyer of a factor of production can have monopsony power, seller of the factor input can also have monopoly power. Example is the labour union which act as a monopolist and determine labour employment and wage rate. Figure 9 shows such case.

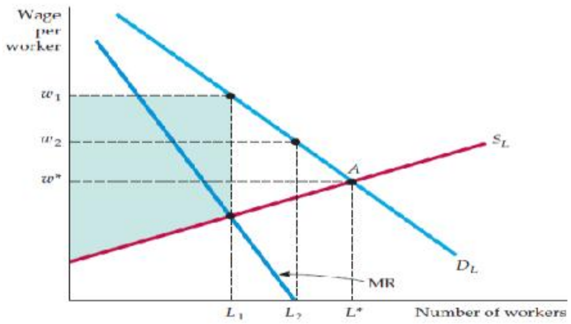

Figure 9 (Source : Pindyck and Rubinfeld, Microeconomics)

When a labour union is a monopolist, it chooses among points on the buyer's labour demand curve (D_L) which is nothing but firm's MRP_L curve. From the viewpoint of monopolist seller, demand curve for labour will act as average revenue (*AR*) curve. So the seller will choose among different points on the labour demand curve D_L. It can choose maximum employment L^* at the competitive wage w^*. Again, monopolist union may choose employment L_1 which is determined by the intersection of *MR* (corresponding to $AR = D_L$) with labour supply curve S_L. In this case union will receive highest possible wage w_1. Lastly, union may also choose w_2 and L_2 on the basis of demand curve for labour D_L. This wage w_2 will maximize total wage paid to the worker, but fetch zero marginal revenue (*MR)*.

Reference: (1) A. Koutsoyannis, Modern Microeconomics.

(2) R.Pindyck and D. Rubinfeld, Microeconomics.

Model Questions: Chapter 5: Choice under uncertainty and markets with asymmetric information

Multiple Choice Questions (MCQ)

Q1. Consider a lottery with three possible outcomes (i) Rs. 100 will be received with probability 0.1 (ii) Rs. 50 will be received with probability 0.2 and (iii) Rs. 10 will be received with probability 0.7. What will be expected value of the lottery?

A) Rs.25
B) Rs. 27
C) Rs. 30
D) None of the above

Answer : B)

Q2. Suppose Natasha's utility function is given by $U(Y) = Y^{0.5}$ where Y is her monthly income. Here Natasha's utility function shows she will be

a) Risk averse
b) Risk neutral
c) Risk lover
d) None of the above

Answer: **a)**

Q3. When a person is known to be risk neutral his utility function will be

(a) Concave
(b) Convex
(c) Linear
(d) Rectangular Hyperbola

Answer: c)

Q4. Any compound lottery is given as $\{L_{1,}L_2,L_3 \ : 1/3,\ 1/3,1/3\}$ where L_1= (1,0,0), L_2= (1/4,3/8,3/8), L_3= (1/4,3/8,3/8). Corresponding reduced form lottery L is given as

(a) (1/2,1/4,1/4)
(b) (1/2,0,1/2)
(c) (1/4,1/2,1/4)
(d) (1/4,1/4,1/2)

Answer : a)

Q5. Comparing the relationship between utility level, expected income and variability of income of the consumer from different available jobs following conclusion is true

(a) Utility is negatively related to expected income and positively to variability of income
(b) Utility is positively related to expected income and negatively to variability of income

(c) Utility is positively related to both expected income and variability of income
(d) Utility is positively related to expected income but uncorrelated to variability of income.

Answer: b)

Q6. Comparing the relationship between risk aversion and preference,

(a) Indifference curves are relatively steep if consumer is highly risk averse
(b) Indifference curves are relatively less steep when consumer is highly risk averse
(c) Indifference curves are of usual shape when consumer is highly risk averse
(d) None of the above

Answer: a)

Q7. Suppose Seema is presently earning a certain income of Rs 40,000 that will continue in the next year. She is offered a chance to take a new job that offers a 0.6 probability of earning Rs.44,000 and a 0.4 probability of earning Rs 33,000. It is also known that Seema is a risk averse person. So her risk premium in the present context will be

(a) Rs 600
(b) Rs 1000
(c) Rs 400
(d) Rs 200

Answer: c)

Q8. The objective of diversification of portfolios is

(a) To obtain higher return from investments
(b) To reduce risk of investments
(c) To obtain more information regarding investments
(d) To attain leadership in investments

Answer: b)

Q9. Suppose there is an assignment of numbers $u_1, u_2, \ldots, u_n$ to the n outcomes (1,2,3,……,n) such that for any simple lottery L = $(p_1, p_2, p_3, \ldots, p_n)$ where p_i is the probability of occurrence of u_i ,we can write

$u(L) = u_1 p_1 + u_2 p_2 + \ldots + u_n p_n$. This function u(L) is called

(a) Utility function
(b) Expected Utility Function
(c) vNM utility function
(d) both b and c

Answer: d)

Q10. Suppose $U = \sqrt{W}$ is the vNM utility function of a consumer where *W* stands for the wealth of the consumer. It is also given that initial wealth of the consumer is 36. Now the consumer is facing the gamble of winning wealth 13 with probability 2/3 and losing wealth 11 with probability 1/3.

(a) The consumer will accept the gamble

(b) The consumer will not accept the gamble.
(c) The consumer will be indifferent between accepting or not accepting the gamble
(d) The consumer's choice cannot be determined.

Answer: (a).

DESCRIPTIVE QUESTION :

1(a). Consider a lottery with three possible outcome

(i) Rs 125 will be received with probability 0.2
(ii) Rs 100 will be received with probability 0.3
(iii) Rs 50 will be received with probability 0.5

a. What is the expected value of the lottery?
b. What is the variance of the outcome?

(b) A risk averse person is offered a choice between a gamble of paying Rs 1000 with probability 0.25 and Rs. 100 with probability 0.75 or a certain payment of Rs. 325. Which one should he choose and why?

2. Following Von Neumann and Morgenstern utility theory, explain the decision making pattern of Risk Lover, Risk Neutral and Risk Averter under uncertainty. How would the risk averter be willing to choose a riskier option?

3. What are the ways in which consumer can reduce risks? Construct a model to explain an investor's choice problem between risk and return while the investor divides his funds between treasury bill (risk free) and stocks (risky asset)

4. Suppose that two investments have the same 3 payoffs, but the probability associated with each payoff differs as illustrated in the table below

Payoff	Probability (Investment A)	Probability (Investment B)
Rs. 300	0.10	0.30
Rs 250	0.80	0.40
Rs 200	0.10	0.30

a. Find the expected return and the standard deviation of each investment.
b. Aloka has the utility function U=5I where I denotes the payoff. Which investment will she choose?
c. Sashi has the utility function $U=\sqrt{5I}$. Which investment will she choose?
d. Indu has the utility function $U=5I^2$. Which investment will she choose?

5. Define a lottery. State the axioms of a lottery space. Define vNM utility function.

6. Define Arrow Pratt measure of risk aversion and categorise how the utility function of the consumer is related to the nature of risk aversion? What is risk premium?

7. Define adverse selection. "With asymmetric information, low quality car drives out good quality car from the market"– justify the statement with Akerlof's used car model.

8. Write short notes on the following :

a) Risk Premium b) Simple lottery vs. Compund lottery c) Risk aversion and utility functions
d) Moral Hazard

9. A family farm has initial wealth Rs 2,50,000. Owner of the farm has two options. Either he can sit idle and invest previous year's income of Rs 2,00,000 at the interest rate 5% or he can plant wheat. Planting costs Rs 2,00,000 with a sixth month time to harvest. If there is a rain, planting wheat will earn Rs 5,00,000 as revenue but if there is a draught planting will earn only Rs 50,000 as revenue. Probability of rain is 0.70 and draught is 0.30. Utility function of the family is given as $U(W)=\sqrt{W}$ where W stands for wealth.

Which option the farm owner will choose? Explain.

10. A moderately risk-averse investor has 50 percent of her portfolio invested in stocks and 50 percent in risk free treasury bills. Show how each of the following events will affect the investors budget line and proportion of stocks in her portfolio:

a. The standard deviation of the return on the stock market increases, but the expected return on the stock market remains same.

b. The expected return on the stock market increases, but the standard deviation of the stock market remains same.

Chapter 7: Factor Pricing Under different market structure

MCQ (Multiple Choice Questions)

Q1: When factor market (labour) is perfectly competitive, firm's supply curve of labour will be

a. Upward rising
b. Downward sloping
c. Parallel to horizontal axis
d. Parallel to vertical axis

Ans: c)

Q2. Factor demand curve is called derived demand because

a. It is derived from production function
b. It is derived from nature of competition in the output market
c. It is derived from competition in factor market
d. None of the above

Ans: b)

Q3. Market supply curve for labour will be

a. Upward rising
b. Downward sloping
c. Backward bending
d. Parallel to horizontal axis

Ans: c)

Q4. When factor market is competitive but product market is monopolistic

a. $VMP_L = MRP_L$
b. $VMP_L > MRP_L$
c. $VMP_L < MRP_L$
d. None of the above

Ans: b)

Q5. Labour demand curve of the firm under several variable inputs is

a. Steeper than that under single variable input
b. Flatter than that under single variable input
c. Equal slope with that under single variable input
d. Unrelated to that under single variable input

Answer: b)

Q6. When both factor market and product market are perfectly competitive

a. VMP_L is the labour demand curve of the firm
b. MRP_L is the labour demand curve of the firm
c. Both a and b
d. Neither a nor b

Answer: c)

Q7. Industry demand curve for labour is the horizontal sum of firm labour demand when

a. Product price is fixed
b. Product price is variable
c. Input price is variable,
d. None of the above

Ans: a)

Q8. Monopsony power in factor market indicates

a. Single seller of factor
b. Single buyer of factor
c. Multiple
d. seller of factor
e. Multiple buyer of factor

Answer: b)

Q9. Monopolistic explotation of labour arises when

a. Factor market is competitive but product market is monopolistic
b. Factor market is monopsonistic but product market is competitive
c. Both factor market and product market are monopolistic
d. Factor market and product market is are competitive

Ans: a)

Q10. Bilateral monopoly occurs in the factor market when

a. seller is monopolist and buyer is monopsonist
b. both buyer and seller in the market are competitive
c. seller is monopolist but buyer is competitive
d. seller is competitive and buyer is monopsonist

Ans: a)

Descriptive Question

1. Show how the equilibrium price and quantity of a factor are determined when there is perfect competition in both commodity and factor market. Why input demand is called derived demand?

2. Derive the demand curve of a factor input (labour) under a competitive market (commodity and factor), when several inputs are variable .

3. Show how individual labour supply curve is derived using indifference curve analysis

4. Suppose factor market is perfectly competitive and product market is monopolistic. How demand curve for labour is determined for individual firm? Can market demand curve for labour be obtained by summation of individual labour demand? Explain with diagram.

5. How labour demand and wage rate are determined in the factor market with monopsonist buyer but competitive seller? Is this wage is equivalent to the wage under competitive factor and product market? Explain

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Factor Pricing

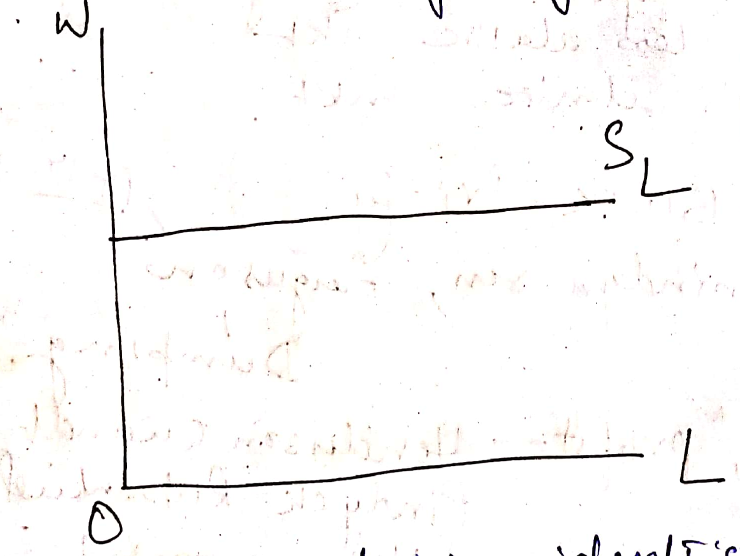
1) D^d curve for a single variable factor in P.C mkt.

single variable factor – labour.

Assumption –

① Large no. of firms in factor mkt.
② Labour are not unionised.

Under these conditions the firms face a perfectly elastic SS curve of labour i.e the firm can employ as much labour as it wants at the going market wage.



Now, identical firms, identical cost.
PC – labour, PC – commodity.
commodity price is given

$$\pi = P_x q - C(q)$$

$q = f(l)\bar{K}$

$\pi = \bar{P}_x f(L) - (\bar{w}L + F)$

$\frac{d\pi}{dL} = \bar{P}_x f'(L) - \bar{w} = 0 \rightarrow \bar{P}_x MP_L = \bar{w}$ or $VMP_L = \bar{w}$ [Money value of Marginal product of L valued in terms of the mkt price of commodity]

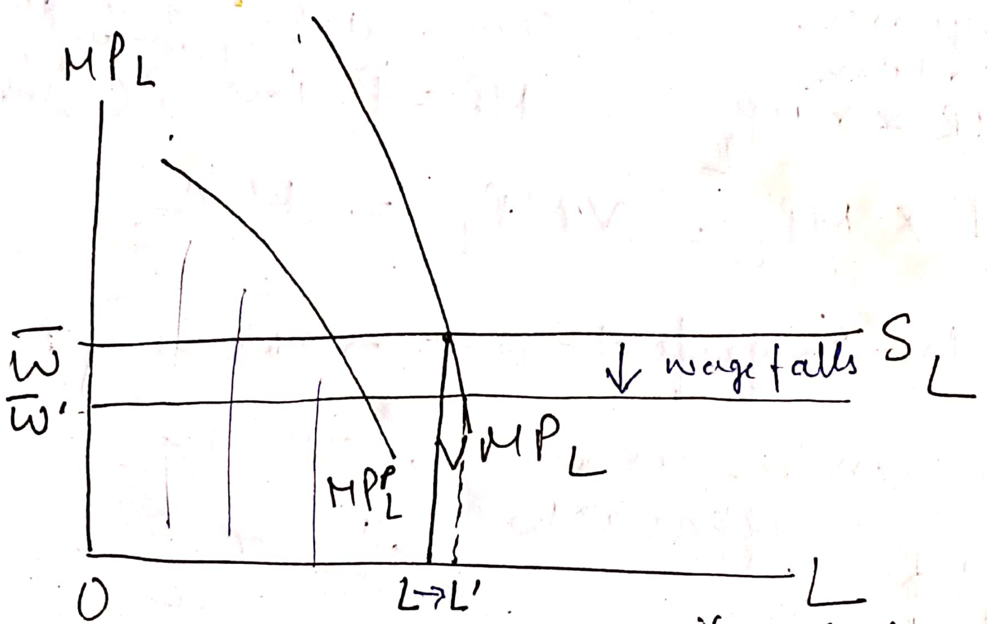
$MP_L \,/\, f'(L) = \frac{\bar{w}}{\bar{P}}$

provided SOC fulfilled,

$\frac{d^2\pi}{dL^2} < 0 \quad \bar{P}_x f''(L) < 0$

or $f''(L) < 0$.

MP_L should be downward sloping.



We sum. VMP_L shift if MP_L ↑ or $\bar{P}$ rises.

It is downward sloping part of VMP_L curve which is demand curve for Labour.

So, in PC, labour & comm mkt are PC, dd curve of firm is downward sloping part of VMP$_L$ curve. This is according to marginalist principle bcz a profit maximising entrepreneur will demand that amount of Labour for which the marginal revenue derived from the extra unit of employment is equal to the marginal cost of this extra unit of employment.

$MRP_L = MC$

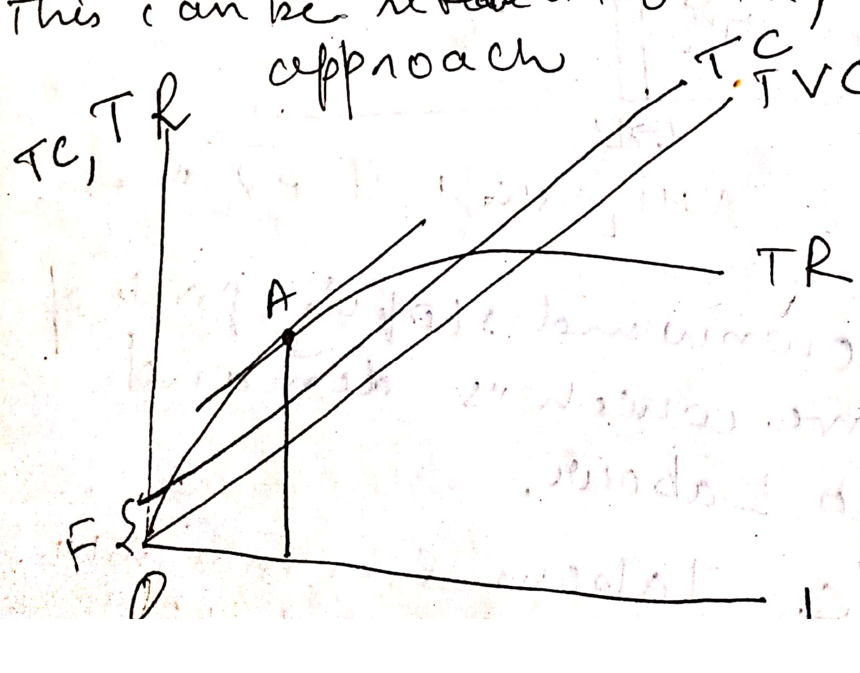
marginal rev times $MR \times MP_L$

wage rate itself. Const. extra unit employ wage not chg. $MR = P$ in PC mkt.

$P \times MP_L = VMP_L = MC$.

[in Imperfect mkt → not so]

This can be revealed in TR, TC approach



[Imperfect mkt]

profit max^m at A. ∵ parallel region.

$\frac{dR}{dL}$ ⇒ L changes by 1 unit when L chg by 1 unit.

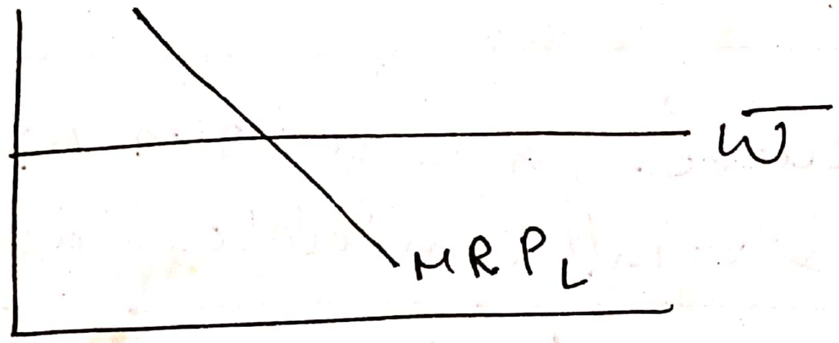
$= \frac{dR}{dq} \times \frac{dq}{dL} = MR \times MP_L$

(slope of TR = slope of TC)

$\therefore MRP_L = \overline{w}$

So, it is MRP_L.

imperfect mkt.



MRP_L is labour dd curve. it

21. Pricing of Factors of Production and Income Distribution

The subject-matter of the theory of income distribution is the study of the determination of the shares of the factors of production in the total output produced in the economy over a given time period. If, for simplicity, we assume that there are two factors of production, labour and capital, their shares are defined as follows

$$[\text{share of labour}] = \frac{w \cdot L}{X}$$

$$[\text{share of capital}] = \frac{r \cdot K}{X}$$

where w = wage rate
r = rental of capital
L = quantity of labour employed
K = quantity of capital employed
X = value of output produced in the economy.

The factor shares depend on the state of technology which defines the production function, and on the relative factor prices.

In Chapter 2 we saw that the production function defines the technically efficient combinations of factors for the production of various levels of output. These combinations define factor intensities, which are measured by the capital–labour ratio (K/L). The factor intensity in the production of any commodity depends on the substitutability of factors. We saw in Chapter 2 that a measure of the degree of substitution of factors is the elasticity of substitution defined as

$$\sigma = \frac{\%\ \text{change of}\ K/L}{\%\ \text{change of}\ MRTS_{L,K}} = \frac{d(K/L)/(K/L)}{d(MRTS)/(MRTS)}$$

In Chapter 2 we also saw that the choice by a firm of one among all the technically efficient combinations of factors depends on the relative prices of these factors. The equilibrium of the firm is defined by the tangency of isoquants and isocost lines, that is, by equating the slope of the isoquants ($MRTS_{L,K}$) to the slope of the isocost lines (w/r).

$$MRTS_{L,K} = \frac{w}{r}$$

To the above determinants of income distribution we must add technological progress. Technical progress usually changes the factor intensity in the production of the various goods. For example, if the technological progress is of the capital-deepening type, there will be some substitution of capital for labour, which will lead to an increase in the share of capital to the total product of the economy.

In summary we may write

$$\begin{bmatrix}\text{income}\\ \text{distribution}\end{bmatrix} = f\left[\begin{pmatrix}\text{production}\\ \text{function}\end{pmatrix}, \begin{pmatrix}\frac{w}{r}\end{pmatrix}, \begin{pmatrix}\text{technical}\\ \text{progress}\end{pmatrix}\right]$$

There is a strong interrelationship between the three determinants of income distribution, arising from the fact that technology and technical progress affect the factors' demand and supply which determine their prices.

In the first section of this chapter we will examine the determination of the prices of variable productive resources in perfectly competitive markets, as well as in markets with various degrees of imperfection. In the second section we will discuss the relation between the elasticity of substitution of productive resources and their shares, and the effects of technical progress on income distribution. In the final section we will discuss some additional topics related to income distribution, namely, (*a*) the pricing of factors in fixed supply and the associated concepts of economic rent and quasi-rent, (*b*) the main factors responsible for wage differentials, (*c*) the problem of 'exhaustion' of the total output in an economy by factor payments.

I. FACTOR PRICING

The mechanism of determination of factor prices does not differ fundamentally from that of prices of commodities. Factor prices are determined in markets under the forces of demand and supply. The difference lies in the determinants of the demand and supply of productive resources.

In the nineteenth century economists classified factor inputs into four groups: land, labour, capital and entrepreneurship. The prices of these factors were called rent, wage, interest and profit respectively, and each one was examined by a separate body of theory. Since, however, there are many common factors underlying the determination of the price of inputs, a general framework can be developed for analysing the price mechanism of any productive resource. Thus, the theory to be developed in this section, will be presented in general terms, so that it is applicable to all factors of production. Given that labour is the most important input, we will usually speak of 'the demand for labour' or, 'the supply of labour'. But the reader should interpret such expressions as implying 'the demand for a productive factor' and 'the supply of a productive factor'.

In this section we will be concerned with the price of variable factors. The determination of the price of factors in fixed supply will be examined in the third section.

We will first examine the determination of factor prices in perfectly competitive product and input markets. Subsequently we will relax the assumption of perfectly competitive markets and we will discuss factor pricing in markets with various degrees of imperfection.

A. FACTOR PRICING IN PERFECTLY COMPETITIVE MARKETS

In this part we will develop the so-called *marginal productivity theory of distribution*. It takes its name from the fact that, in perfectly competitive product and input markets, factors are paid the value of their marginal physical product (see below).

We said earlier that the price of a factor, w, is determined by its total demand and supply schedules. The total demand is the sum (aggregate) of the demands of individual firms for the productive factor. Similarly, the total supply of a factor is the sum of the supplies by the individual owners of the factor.

Following the methodology of earlier chapters we will develop first the demand for labour by a single firm. The aggregate demand will then be derived from the summation of the individual demands. The same approach will be adopted for the market supply. We will first derive the supply of labour by an individual consumer. The aggregate supply of labour will then be derived from the summation of the individual supply curves.

I. The demand for labour in perfectly competitive markets

We will examine the demand for labour in two cases: (i) when labour is the only variable factor of production, (ii) when there are several variable factors.

(i) *Demand of a firm for a single variable factor*

The following assumptions underlie our analysis:

(*a*) A single commodity X is produced in a perfectly competitive market. Hence P_x is given for all firms in the market.

(*b*) The goal of the firm is profit maximisation.

(*c*) There is a single variable factor, labour, whose market is perfectly competitive. Hence the price of labour services, $\bar{w}$, is given for all firms. This implies that the supply of labour to the individual firm is perfectly elastic. It can be denoted by a straight line through $\bar{w}$ parallel to the horizontal axis (figure 21.1). At the going market wage rate the firm can employ (hire) any amount of labour it wants.

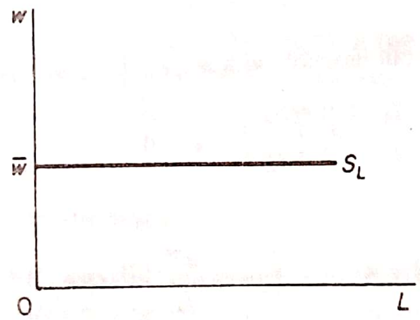


Figure 21.1

(*d*) Technology is given. The relevant section of the production function is shown in figure 21.2. The slope of the production function is the marginal physical product of labour

$$\frac{dX}{dL} = MPP_L$$

The MPP_L declines at higher levels of employment, given the law of variable proportions. (See Chapter 2.) If we multiply the MPP_L at each level of employment by the given price of the output, $\bar{P}_x$, we obtain the *value-of-marginal-product curve* VMP_L (figure 21.3). This curve shows the value of the output produced by an additional unit of labour employed.

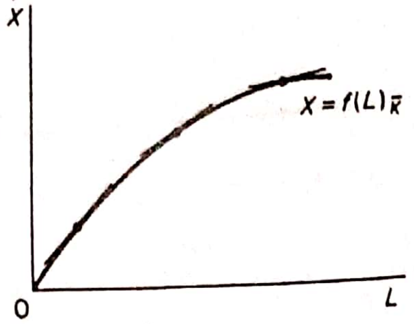


Figure 21.2

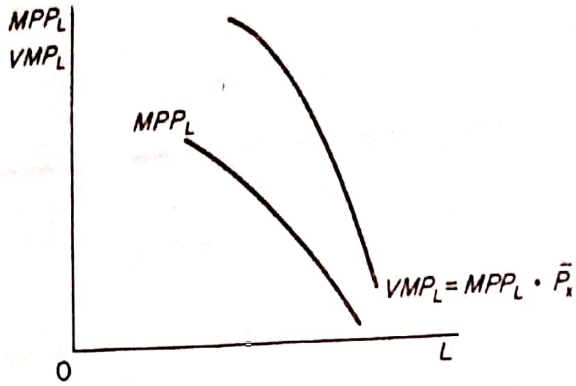


Figure 21.3

The firm, being a profit maximiser, will hire a factor as long as it adds more to total revenue than to total cost. Thus a firm will hire a resource up to the point at which the last unit contributes as much to total cost as to total revenue, because total profit cannot be further increased. In other words the condition of equilibrium of a profit maximiser in the labour market is

$$MC_L = VMP_L$$

where MC_L = marginal cost of labour,

or $$\bar{w} = VMP_L$$ [1]

given that $MC_L = \bar{w}$.

In figure 21.4 the equilibrium of the firm is denoted by e. At the market wage rate $\bar{w}$ the firm will maximise its profit hiring l^* units of labour. This is so because to the left

[1] *Formal derivation of the equilibrium of the firm*

The production function is

$$X = f(L)_{\bar{K}}$$

The total cost consists of the variable cost $\bar{w} \cdot L$ and the fixed cost F

$$C = \bar{w} \cdot L + F$$

The revenue of the firm is $R = \bar{P}_x \cdot X = \bar{P}_x \cdot [f(L)]$. The firm wants to maximise its profit

$$\Pi = R - C$$
$$\Pi = \bar{P}_x \cdot [f(L)] - (\bar{w} \cdot L + F)$$

Setting the first derivative of the profit function with respect to labour equal to zero we obtain

$$\frac{d\Pi}{dL} = \bar{P}_x \cdot \left(\frac{dX}{dL}\right) - \bar{w} = 0$$

Rearranging

$$\bar{P}_x \cdot (MPP_L) = \bar{w} \qquad \left(\text{given } \frac{dX}{dL} = MPP_L\right)$$

or

$$VMP_L = \bar{w}$$ Q.E.D.

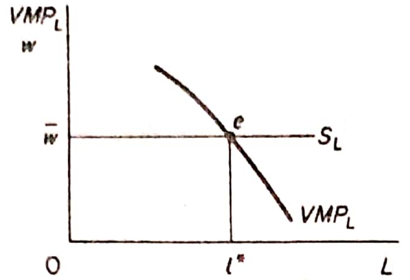


Figure 21.4

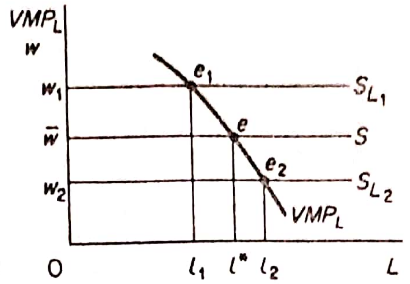


Figure 21.5

of l^* each unit of labour costs less than the value of its product ($VMP_L > \bar{w}$), hence the profit of the firm will be increased by hiring more workers. Conversely to the right of l^* the $VMP_L < \bar{w}$, and hence profits are reduced. It follows that profits are at a maximum when $VMP_L = \bar{w}$.

If the market wage is raised to w_1, the firm will reduce its demand for labour to l_1 (figure 21.5) in order to maximise its profit (at e_1 in figure 21.5 $w_1 = VMP_L$). Similarly, if the wage falls to w_2, the firm will maximise its profit by increasing its employment to l_2.

It follows from the above analysis that the demand curve of a firm for a single variable factor is its value-of-marginal-product curve.

As an illustration of the above discussion consider the following numerical example.

Assume a production process which involves a fixed amount of machinery (e.g. ten machines) giving rise to a total fixed cost of £50, and labour which is the only variable factor. The wage rate is £40 and the price of the commodity produced is £10. The production function is specified by the information of the first four columns of table 21.1. Column 6 shows total revenue ($= X \cdot \bar{P}_x$), column 10 includes total variable cost ($= L \cdot w$). Finally column 12 shows the profit of the firm ($\Pi = R - TVC - FC$).

The demand for labour which maximises the profit of the firm can be determined either by using the total revenue and total cost curves, or by using the VMP_L schedule and the given wage rate, which defines the supply of labour to the individual firm.

1. *The total revenue–total cost approach*

Profit is at a maximum when the difference between total revenue and total cost is greatest. In our example this occurs when nine units of labour are used. This solution, therefore, corresponds to the profit-maximising position of the firm.

The total revenue–total cost approach is shown in figure 21.6. From this figure we see that the maximum distance between the two curves occurs when the firm employs nine units of labour. At this level of employment the slope of the total revenue and the total cost curves are equal. The slope of the revenue curve is the marginal revenue per additional unit of labour, and the slope of the total cost curve is the wage rate, which in perfectly competitive markets is equal to the marginal cost of labour. Thus the condition for the equilibrium of the firm in the factor market is

$$MRP_L = w = MC_L$$

Given that

$$MRP_L = \frac{\partial R}{\partial L} = \frac{\partial(X \cdot \bar{P}_x)}{\partial L} = \bar{P}_x \cdot \frac{\partial X}{\partial L} = \bar{P}_x \cdot (MPP_L)$$

Table 21.1 Data for the derivation of the demand for labour by a firm

The production function				Costs, revenues, profit							
Units of fixed capital K	*Units of labour* L	*Total output (units)* X	*Marginal physical product of labour* MPP_L	*Price of product* P_x £	*Total revenue* $R = X \cdot \bar{P}_x$ £	*Value of marginal physical product of labour* VMP_L (£)	*Total fixed cost* £	*Wage rate* $\bar{w}$ £	*Total variable cost* $TVC = \bar{w} \cdot L$ £	*Total cost* $TC = TVC + FC$	*Profit* $\Pi = R - TC$ £
10	0	0	—	10	0	0	50	40	0	50	−50
10	1	20	20	10	200	200	50	40	40	90	110
10	2	38	18	10	380	180	50	40	80	130	250
10	3	54	16	10	540	160	50	40	120	170	370
10	4	68	14	10	680	140	50	40	160	210	420
10	5	80	12	10	800	120	50	40	200	250	550
10	6	90	10	10	900	100	50	40	240	290	610
10	7	98	8	10	980	80	50	40	280	330	650
10	8	104	6	10	1040	60	50	40	320	370	670
10	9	108	4	10	1080	40	50	40	360	410	670
10	10	110	2	10	1100	20	50	40	400	450	650

Note. Two levels of employment ($l = 8$ and $l = 9$) give the same (maximum) level of profit (= £670). This is due to the fact that in our numerical example we work with finite changes instead of the required infinitesimally small changes.

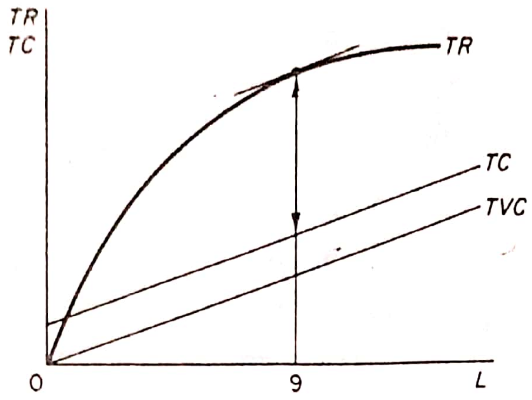


Figure 21.6

and by definition

$$\bar{P}_x \cdot (MPP_L) = VMP_L$$

we may write the equilibrium condition as

$$VMP_L = w$$

which is the same result as the one we reached above.[1]

2. The VMP$_L$ *approach*

In figure 21.7 we show the VMP_L of our numerical example. The supply of labour to the individual firm is the straight line S_l passing through the given wage rate of \$40. The two curves intersect at point e, which defines the demand for labour ($l = 9$) at which the profit of the firm is at a maximum.

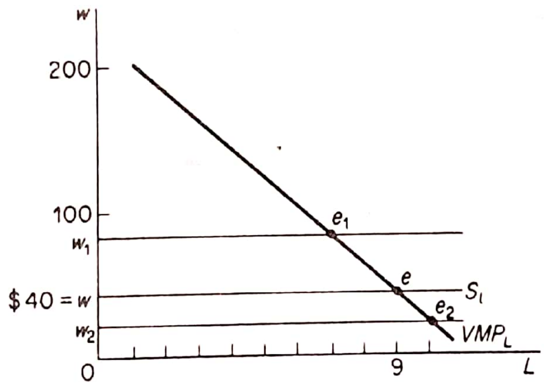


Figure 21.7

The firm is in equilibrium by equating the VMP_L to the market wage rate. If the market wage rises, the equality of w_1 and VMP_L occurs to the left of e. Conversely if the wage rate falls to w_2 the equality with the VMP_L curve occurs to the right of e. Thus the value-of-marginal-product curve is the demand curve for labour of the individual firm.

[1] Note that in perfectly competitive markets $MRP_L = VMP_L$. This is not true for imperfect product markets. (See page 541 below.)

landowners higher rental income than others, and some capital owners greater profit than others? Why, in particular, do computer programmers earn more than gas station attendants?

The answers to these questions, like most in economics, hinge on supply and demand. The supply and demand for labor, land, and capital determine the prices paid to workers, landowners, and capital owners. To understand why some people have higher incomes than others, therefore, we need to look more deeply at the markets for the services they provide. That is our job in this and the next two chapters.

factors of production
the inputs used to produce goods and services

This chapter provides the basic theory for the analysis of factor markets. As you may recall from Chapter 2, the **factors of production** are the inputs used to produce goods and services. Labor, land, and capital are the three most important factors of production. When a computer firm produces a new software program, it uses programmers' time (labor), the physical space on which its offices are located (land), and an office building and computer equipment (capital). Similarly, when a gas station sells gas, it uses attendants' time (labor), the physical space (land), and the gas tanks and pumps (capital).

In many ways factor markets resemble the markets for goods and services we analyzed in previous chapters, but they are different in one important way: The demand for a factor of production is a *derived demand*. That is, a firm's demand for a factor of production is derived from its decision to supply a good in another market. The demand for computer programmers is inseparably linked to the supply of computer software, and the demand for gas station attendants is inseparably linked to the supply of gasoline.

In this chapter, we analyze factor demand by considering how a competitive, profit-maximizing firm decides how much of any factor to buy. We begin our analysis by examining the demand for labor. Labor is the most important factor of production, because workers receive most of the total income earned in the U.S. economy. Later in the chapter, we will see that our analysis of the labor market also applies to the markets for the other factors of production.

The basic theory of factor markets developed in this chapter takes a large step toward explaining how the income of the U.S. economy is distributed among workers, landowners, and owners of capital. Chapter 19 builds on this analysis to examine in more detail why some workers earn more than others. Chapter 20 examines how much income inequality results from the functioning of factor markets and then considers what role the government should and does play in altering the income distribution.

The Demand for Labor

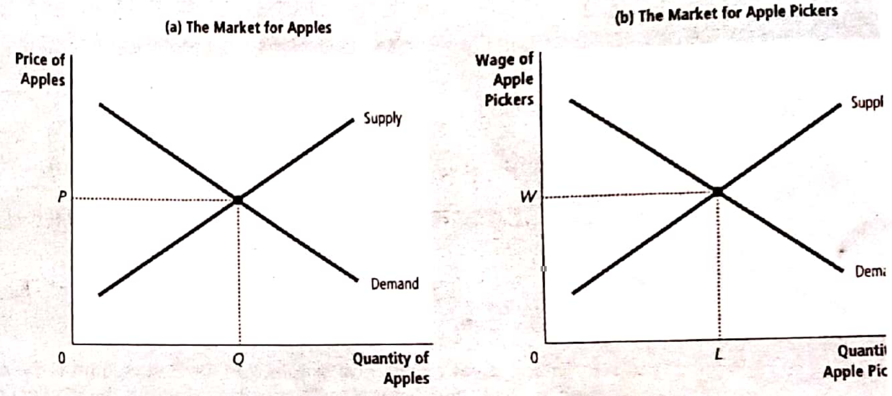
Labor markets, like other markets in the economy, are governed by the forces of supply and demand. This is illustrated in Figure 1. In panel (a), the supply and demand for apples determine the price of apples. In panel (b), the supply and demand for apple pickers determine the price, or wage, of apple pickers.

As we have already noted, labor markets are different from most other markets because labor demand is a derived demand. Most labor services, rather than being final goods ready to be enjoyed by consumers, are inputs into the production of other goods. To understand labor demand, we need to focus on the firms that hire the labor and use it to produce goods for sale. By examining the link between the production of goods and the demand for labor to make those goods, we gain insight into the determination of equilibrium wages.

Figu

The Versatility of S and Demand

The basic tools of supply and demand apply to goods and to labor services. Panel (a) shows how the supply and demand for apples determine the price of apples. Panel (b) shows how the supply and demand for apple pickers determine the wage of apple pickers.



The Competitive Profit-Maximizing Firm

Let's look at how a typical firm, such as an apple producer, decides what quantity of labor to demand. The firm owns an apple orchard and each week must decide how many apple pickers to hire to harvest its crop. After the firm makes its hiring decision, the workers pick as many apples as they can. The firm then sells the apples, pays the workers, and keeps what is left as profit.

We make two assumptions about our firm. First, we assume that our firm is *competitive* both in the market for apples (where the firm is a seller) and in the market for apple pickers (where the firm is a buyer). A competitive firm is a price taker. Because there are many other firms selling apples and hiring apple pickers, a single firm has little influence over the price it gets for apples or the wage it pays apple pickers. The firm takes the price and the wage as given by market conditions. It only has to decide how many apples to sell and how many workers to hire.

Second, we assume that the firm is *profit maximizing*. Thus, the firm does not directly care about the number of workers it has or the number of apples it produces. It cares only about profit, which equals the total revenue from the sale of apples minus the total cost of producing them. The firm's supply of apples and its demand for workers are derived from its primary goal of maximizing profit.

The Production Function and the Marginal Product of Labor

To make its hiring decision, the firm must consider how the size of its workforce affects the amount of output produced. In other words, it must consider how the number of apple pickers affects the quantity of apples it can harvest

1
the Competitive
Decides How
Labor to Hire

Labor L	Output Q	Marginal Product of Labor $MPL = \Delta Q/\Delta L$	Value of the Marginal Product of Labor $VMPL = P \times MPL$	Wage W	Marginal Profit $\Delta Profit = VMPL - W$
0 workers	0 bushels				
		100 bushels	$1,000	$500	$500
1	100				
		80	800	500	300
2	180				
		60	600	500	100
3	240				
		40	400	500	−100
4	280				
		20	200	500	−300
5	300				

and sell. Table 1 gives a numerical example. In the first column is the number of workers. In the second column is the quantity of apples the workers harvest each week.

production function the relationship between quantity of inputs used to make a good and the quantity of output of that good

These two columns of numbers describe the firm's ability to produce. Recall that economists use the term **production function** to describe the relationship between the quantity of the inputs used in production and the quantity of output from production. Here the "input" is the apple pickers and the "output" is the apples. The other inputs—the trees themselves, the land, the firm's trucks and tractors, and so on—are held fixed for now. This firm's production function shows that if the firm hires 1 worker, that worker will pick 100 bushels of apples per week. If the firm hires 2 workers, the 2 workers together will pick 180 bushels per week. And so on.

Figure 2 graphs the data on labor and output presented in Table 1. The number of workers is on the horizontal axis, and the amount of output is on the vertical axis. This figure illustrates the production function.

One of the *Ten Principles of Economics* introduced in Chapter 1 is that rational people think at the margin. This idea is the key to understanding how firms decide what quantity of labor to hire. To take a step toward this decision, the third column in Table 1 gives the **marginal product of labor**, the increase in the amount of output from an additional unit of labor. When the firm increases the number of workers from 1 to 2, for example, the amount of apples produced rises from 100 to 180 bushels. Therefore, the marginal product of the second worker is 80 bushels.

marginal product of labor the increase in the amount of output from an additional unit of labor

Notice that as the number of workers increases, the marginal product of labor declines. That is, the production process exhibits **diminishing marginal product**. At first, when only a few workers are hired, they can pick the low-hanging fruit. As the number of workers increases, additional workers have to climb higher up the ladders to find apples to pick. Hence, as more and more workers are hired, each additional worker contributes less to the production of apples. For this reason, the production function in Figure 2 becomes flatter as the number of workers rises.

diminishing marginal product the property whereby the marginal product of an input declines as the quantity of the input increases

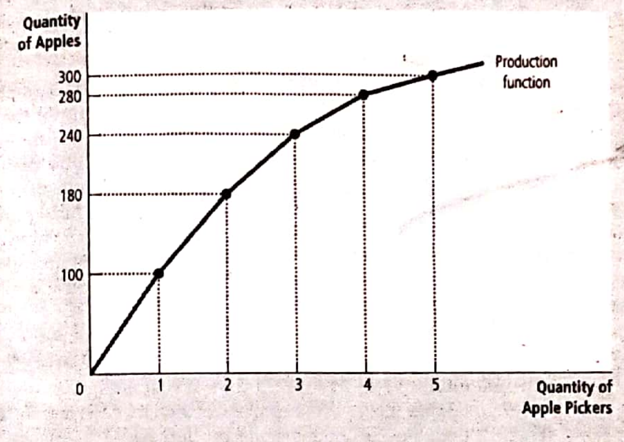


Figure 2

The Production Function The production function is the relationship between the inputs into production (apple pickers) and the output from production (apples). As the quantity of the input increases, the production function gets flatter, reflecting the property of diminishing marginal product.

The Value of the Marginal Product and the Demand for Labor

Our profit-maximizing firm is concerned more with money than with apples. As a result, when deciding how many workers to hire to pick apples, the firm considers how much profit each worker would bring in. Because profit is total revenue minus total cost, the profit from an additional worker is the worker's contribution to revenue minus the worker's wage.

To find the worker's contribution to revenue, we must convert the marginal product of labor (which is measured in bushels of apples) into the *value* of the marginal product (which is measured in dollars). We do this using the price of apples. To continue our example, if a bushel of apples sells for $10 and if an additional worker produces 80 bushels of apples, then the worker produces $800 of revenue.

value of the marginal product the marginal product of an input times the price of the output

The **value of the marginal product** of any input is the marginal product of that input multiplied by the market price of the output. The fourth column in Table 1 shows the value of the marginal product of labor in our example, assuming the price of apples is $10 per bushel. Because the market price is constant for a competitive firm while the marginal product declines with more workers, the value of the marginal product diminishes as the number of workers rises. Economists sometimes call this column of numbers the firm's *marginal revenue product:* It is the extra revenue the firm gets from hiring an additional unit of a factor of production.

Now consider how many workers the firm will hire. Suppose that the market wage for apple pickers is $500 per week. In this case, as you can see in Table 1, the first worker that the firm hires is profitable: The first worker yields $1,000 in revenue, or $500 in profit. Similarly, the second worker yields $800 in additional revenue, or $300 in profit. The third worker produces $600 in additional revenue, or \$100 in profit. After the third worker, however, hiring workers is unprofitable. The fourth worker would yield only \$400 of additional revenue. Because the worker's wage is \$500, hiring the fourth worker would mean a \$100 reduction in profit. Thus, the firm hires only 3 workers.

It is instructive to consider the firm's decision graphically. Figure 3 graphs the value of the marginal product. This curve slopes downward because the marginal product of labor diminishes as the number of workers rises. The figure also includes a horizontal line at the market wage. To maximize profit, the firm hires workers up to the point where these two curves cross. Below this level of employment, the value of the marginal product exceeds the wage, so hiring another worker would increase profit. Above this level of employment, the value of the marginal product is less than the wage, so the marginal worker is unprofitable. Thus, *a competitive, profit-maximizing firm hires workers up to the point where the value of the marginal product of labor equals the wage.*

Having explained the profit-maximizing hiring strategy for a competitive firm, we can now offer a theory of labor demand. Recall that a firm's labor-demand curve tells us the quantity of labor that a firm demands at any given wage. We have just seen in Figure 3 that the firm makes that decision by choosing the quantity of labor at which the value of the marginal product equals the wage. As a result, *the value-of-marginal-product curve is the labor-demand curve for a competitive, profit-maximizing firm.*

What Causes the Labor-Demand Curve to Shift?

We now understand the labor-demand curve: It reflects the value of the marginal product of labor. With this insight in mind, let's consider a few of the things that might cause the labor-demand curve to shift.

The Output Price The value of the marginal product is marginal product times the price of the firm's output. Thus, when the output price changes, the value of the marginal product changes, and the labor-demand curve shifts. An increase in the price of apples, for instance, raises the value of the marginal product of each worker who picks apples and, therefore, increases labor demand from the firms that supply apples. Conversely, a decrease in the price of apples reduces the value of the marginal product and decreases labor demand.

Technological Change Between 1960 and 2009, the output a typical U.S. worker produced in an hour rose by 183 percent. Why? The most important reason is technological progress: Scientists and engineers are constantly figuring out new and better ways of doing things. This has profound implications for the labor market. Technological advance typically raises the marginal product of labor, which in turn increases the demand for labor and shifts the labor-demand curve to the right.

It is also possible for technological change to reduce labor demand. The invention of a cheap industrial robot, for instance, could conceivably reduce the marginal product of labor, shifting the labor-demand curve to the left. Economists call this *labor-saving* technological change. History suggests, however, that most

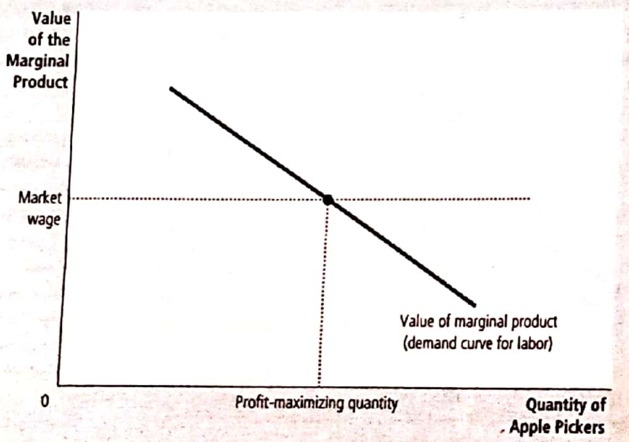


Figure 3

The Value of the Marginal Product of Labor

This figure shows how the value of the marginal product (the marginal product times the price of the output) depends on the number of workers. The curve slopes downward because of diminishing marginal product. For a competitive, profit-maximizing firm, this value-of-marginal-product curve is also the firm's labor-demand curve.

FYI

Input Demand and Output Supply: Two Sides of the Same Coin

In Chapter 14, we saw how a competitive, profit-maximizing firm decides how much of its output to sell: It chooses the quantity of output at which the price of the good equals the marginal cost of production. We have just seen how such a firm decides how much labor to hire: It chooses the quantity of labor at which the wage equals the value of the marginal product. Because the production function links the quantity of inputs to the quantity of output, you should not be surprised to learn that the firm's decision about input demand is closely linked to its decision about output supply. In fact, these two decisions are two sides of the same coin.

To see this relationship more fully, let's consider how the marginal product of labor (*MPL*) and marginal cost (*MC*) are related. Suppose an additional worker costs \$500 and has a marginal product of 50 bushels of apples. In this case, producing 50 more bushels costs \$500; the marginal cost of a bushel is \$500/50, or \$10. More generally, if *W* is the wage, and an extra unit of labor produces *MPL* units of output, then the marginal cost of a unit of output is $MC = W/MPL$.

This analysis shows that diminishing marginal product is closely related to increasing marginal cost. When our apple orchard grows crowded with workers, each additional worker adds less to the production of apples (*MPL* falls). Similarly, when the apple firm is producing a large quantity of apples, the orchard is already crowded with workers, so it is more costly to produce an additional bushel of apples (*MC* rises).

Now consider our criterion for profit maximization. We determined earlier that a profit-maximizing firm chooses the quantity of labor so that the value of the marginal product ($P \times MPL$) equals the wage (*W*). We can write this mathematically as

$$P \times MPL = W.$$

If we divide both sides of this equation by *MPL*, we obtain

$$P = W/MPL.$$

We just noted that *W/MPL* equals marginal cost, *MC*. Therefore, we can substitute to obtain

$$P = MC.$$

This equation states that the price of the firm's output is equal to the marginal cost of producing a unit of output. Thus, *when a competitive firm hires labor up to the point at which the value of the marginal product equals the wage, it also produces up to the point at which the price equals marginal cost.* Our analysis of labor demand in this chapter is just another way of looking at the production decision we first saw in Chapter 14.

The Luddite Revolt

Over the long span of history, technological progress has been the worker's friend. It has increased productivity, labor demand, and wages. Yet there is no doubt that workers sometimes see technological progress as a threat to their standard of living.

One famous example occurred in England in the early 19th century, when skilled knitters saw their jobs threatened by the invention and spread of machines that could produce textiles using less skilled workers and at much lower cost. The displaced workers organized violent revolts against the new technology. They smashed the weaving machines used in the wool and cotton mills and, in some cases, set the homes of the mill owners on fire. Because the workers claimed to be led by General Ned Ludd (who may have been a legendary figure rather than a real person), they were called Luddites.

The Luddites wanted the British government to save their jobs by restricting the spread of the new technology. Instead, the Parliament took action to stop the Luddites. Thousands of troops were sent to suppress the Luddite riots, and the Parliament eventually made destroying machines a capital crime. After a trial in York in 1813, seventeen men were hanged for the offense. Many others were convicted and sent to Australia as prisoners.

Today, the term *Luddite* refers to anyone who opposes technological progress.

The Luddites.

technological progress is instead *labor-augmenting*. Such technological advance explains persistently rising employment in the face of rising wages: Even though wages (adjusted for inflation) increased by 150 percent during the last half century, firms nonetheless increased the amount of labor they employed by 87 percent.

The Supply of Other Factors The quantity available of one factor of production can affect the marginal product of other factors. A fall in the supply of ladders, for instance, will reduce the marginal product of apple pickers and thus the demand for apple pickers. We consider this linkage among the factors of production more fully later in the chapter.

QUICK QUIZ *Define* marginal product of labor *and* value of the marginal product of labor. • *Describe how a competitive, profit-maximizing firm decides how many workers to hire.*

B. FACTOR PRICING IN IMPERFECTLY COMPETITIVE MARKETS

The price of an input, when there are imperfections in the commodity and the factor markets, is determined by the same mechanism as in the case of perfectly competitive markets: demand and supply determine the price of the factor and the level of its employment. However, the determinants of the demand and the supply are different in the case of market imperfections.

We will consider four models with various kinds of imperfections. In the first model we will assume that the firm has monopolistic power in the product market, while the factor market is perfectly competitive. We will next allow for imperfections in the demand for the factor. In particular we will examine the case of a firm which has monopolistic power in the product market and monopsonistic power in the input market. The third model is the case of a bilateral monopoly (the firm has monopsonistic power and the supply is controlled by labour unions. Finally the fourth model refers to the case of a firm which has no monopsonistic power and faces unionised labour supply.

The above models are extensions of the marginal productivity theory of factor pricing and income distribution.

Model A. Monopolistic power in the product market

firm is monopoly in Product Market.

(a) *Demand of a monopolistic firm for a single variable factor*

In this model we assume that the firm uses a single variable factor, labour, whose market is perfect: the wage rate is given and the supply of labour to the individual firm is perfectly elastic. However, the firm has monopolistic power in the market of the commodity it produces. This implies that the demand for the product of the firm is downward-sloping and the marginal revenue is smaller than the price at all levels of output (figure 21.19).

Under these conditions we will show that the demand for labour of an individual firm is *not* the VMP_L curve but the *marginal-revenue-product curve*, defined by multiplying the MPP_L times the marginal revenue of selling the commodity produced:

$$MRP_L = MPP_L \cdot MR_x$$

We may illustrate the derivation of the marginal-revenue-product-of-labour curve, using the numerical example of the previous section, with the difference that the price

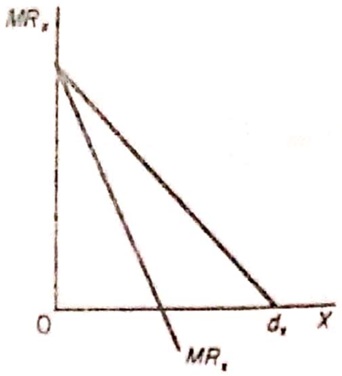


Figure 21.19 Imperfect product market

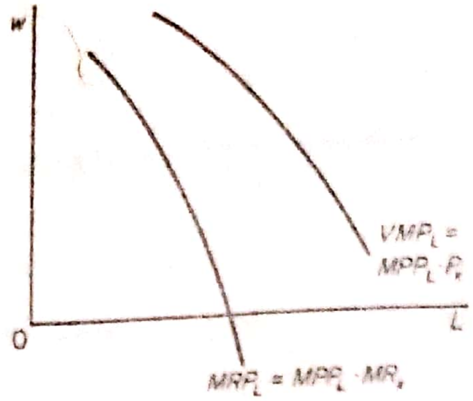


Figure 21.20 Perfect factor market

of the commodity produced declines as output increases. The relevant data are given in table 21.2.

P7[M]

The VMP_L $(= MPP_L \cdot P_x)$ is shown in column 8, and the MRP_L $(= MPP_L \cdot MR_x)$ is shown in column 9. We observe that $VMP_L > MRP_L$. In figure 21.20 the VMP_L curve lies above the MRP_L curve at all levels of employment. This is due to the fact that $P_x > MR_x$ at all levels of output and employment. Both the VMP_L and MRP_L have a negative slope because their components (MPP_L, P_x, and MR_x) decline as output expands and the price of the product falls.

Mathematical derivation of the MRP_L *curve:*

We will show that $MRP_L = MPP_L \cdot MR_x$.

1. Let the demand function for the product be

$$P_x = f_1(Q_x) \tag{1}$$

The total revenue of the firm is

$$TR = P_x \cdot Q_x$$

and the marginal revenue

$$\frac{d(TR)}{dQ_x} = P_x \cdot \frac{dQ_x}{dQ_x} + Q_x \cdot \frac{dP_x}{dQ_x}$$

or

$$MR_x = P_x + Q_x \frac{dP_x}{dQ_x} \tag{2}$$

2. The production function with labour the only variable factor is

$$Q_x = f_2(L)$$

The MPP_L is

$$\frac{dQ_x}{dL} = MPP_L \tag{3}$$

3. By definition the marginal revenue product of labour is the change in total revenue attributable to a unit change in labour

$$MRP_L = \frac{d(TR)}{dL}$$

Table 21.2 Data for the derivation of the MRP_L curve

Units of fixed capital K	*Units of labor* L	*Total output (units)* X	*Marginal physical product of labour* MPP_L	*Price of X* P_x	*Total revenue* $R = P_x \cdot X$	*Marginal revenue* $MR_x = \Delta R/\Delta X$	*Value of marginal product of labour* $VMP_L = MPP_L \cdot P_x$	*Marginal revenue product of labour* $MRP_L = MPP_L \cdot MP_x$	*Wage rate* $\bar{w}$	*Total fixed cost* TFC	*Total variable cost* $TVC = L \cdot w$	*Total cost* C	*Profit* $\Pi = R - C$
10	0	0	—	—	—	—	—	—	—	£50	—	£50	−50
10	1	20	20	£10	£200	£10	£200	£200	£40	50	£40	90	120
10	2	38	18	9.05	344	8	163	144	40	50	80	130	214
10	3	54	16	8.44	456	7	135	112	40	50	120	170	286
10	4	68	14	7.94	540	6	111	84	40	50	160	210	330
10	5	80	12	7.50	600	5	90	60	40	50	200	250	350
10	6	90	10	7.11	640	4	71	40	40	50	240	290	350
10	7	98	8	6.78	664	3	54	24	40	50	280	330	334
10	8	104	6	6.44	670	1	39	6	40	50	320	370	300
10	9	108	4	6.20	670	0	25	0	40	50	360	410	260
10	10	110	2	6.05	666	−2	12	−4	40	50	400	450	216

Note. Two levels of employment ($l = 5$ and $l = 6$) give the same (maximum) level of profit (= £350). This is due to the fact that in our numerical example we work with finite changes instead of the required infinitesimally small changes.

Given $TR = P_x \cdot Q_x$, the derivative of total revenue with respect to L is

$$\frac{d(TR)}{dL} = P_x \cdot \frac{dQ_x}{dL} + Q_x \left[\frac{dP_x}{dQ_x} \cdot \frac{dQ_x}{dL}\right]$$

or

$$MRP_L = \frac{dQ_x}{dL}\left[P_x + Q_x \cdot \frac{dP_x}{dQ_x}\right]$$

But from (3)

$$\frac{dQ_x}{dL} = MPP_L$$

and from (2)

$$\left[P_x + Q_x \cdot \frac{dP_x}{dQ_x}\right] = MR_x$$

Therefore $MRP_L = (MPP_L) \cdot (MR_x)$. Q.E.D.

We will now show that the demand for labour is its marginal-revenue-product curve. Recall that in this model we assume that the labour market is perfectly competitive. Hence the supply of labour to the individual firm is perfectly elastic. This is shown by the horizontal line S_L in figure 21.21, which passes through the market wage $\bar{w}$.

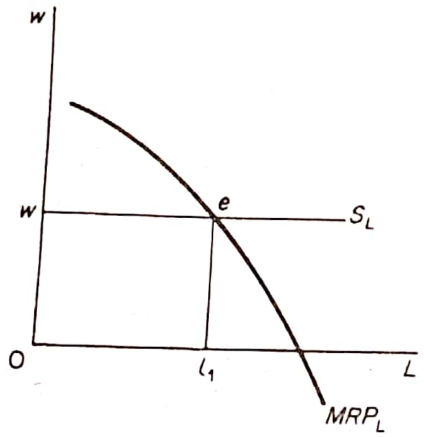


Figure 21.21

The firm, being a profit maximiser, will be in equilibrium at point e, employing l_1, units of labour. At this point

$$MRP_L = MC_L = \bar{w}$$

In other words the firm is in equilibrium in the factor market when it employs units of labour up to the point where the marginal revenue product of labour is equal to its marginal cost. (Recall that $MC_L = \bar{w}$ in a perfect labour market.) e is an equilibrium point because with employment l_1 the firm's profit is maximised. To the left of e a unit of labour adds more to the revenue of the firm than the amount of its cost; hence it pays the firm to increase its employment. Conversely at any point to the right of e an additional unit of labour adds more to total cost than to total revenue. Therefore, a profit-maximising firm (with monopolistic power in the product market) will employ labour up to the point where the marginal revenue product is equal to the wage rate.

The above analysis can be repeated for any given wage rate. Hence if only one variable factor is used, the marginal-revenue-product curve is the monopolist's demand curve for this factor.

In our numerical example the firm maximises its profit by employing six units of labour: at this level of employment $MRP_L = w = £40$, and total profits (£350) are at a maximum.

Formal derivation of the equilibrium of the monopolistic firm

The firm wants to maximise its profit

$$\Pi = R - C = P_x \cdot Q_x - (wL + F)$$

given

$$P_x = f_1(Q_x)$$

and

$$Q_x = f_2(L)$$

The first-order condition for Π max is that the derivative of Π w.r.t. L is equal to zero

$$\frac{d\Pi}{dL} = P_x \cdot \frac{dQ_x}{dL} + Q_x \left[\frac{dP_x}{dQ_x} \cdot \frac{dQ_x}{dL}\right] - w = 0$$

Rearranging we obtain

$$\frac{dQ_x}{dL}\left[P_x + Q_x \cdot \frac{dP_x}{dQ_x}\right] = w$$

We have shown that

$$\frac{dQ_x}{dL} = MPP_L \quad \text{and} \quad \left(P_x + Q_x \frac{dP_x}{dQ_x}\right) = MR_x$$

Therefore

$$(MPP_L) \cdot (MR_x) = w \quad \text{or} \quad MRP_L = w$$

The firm maximises its profit when it employed units of labour up to the point where $MRP_L = w$. Q.E.D.

Given that

$$MR_x = P_x \cdot \left(1 - \frac{1}{e_p}\right)$$

the above equilibrium condition can be written in the form

$$(MPP_L) \cdot (P_x)\left(1 - \frac{1}{e_p}\right) = w$$

which shows the relations among commodity price, factor price, elasticity of demand and the production function.

(b) Demand of a variable factor by a monopolistic firm when several factors are used

When more than one variable factor is used in the production process the demand for a variable factor is *not* its marginal-revenue-product curve, but is formed from points on shifting *MRP* curves.

The analysis is similar to that of the previous section. Suppose that the market price of labour is w_1 and that its marginal revenue product is given by MRP_1 (figure 21.22). The monopolistic firm is in equilibrium at point *A*, employing l_1 units of labour. If the wage rate falls to w_2 the firm would move along its MRP_1 curve, to point *A′*, if other things remained equal. However, other things do not remain equal. The fall in the wage rate has a substitution effect, an output effect and a profit-maximising effect, as in the case of a perfectly competitive firm. The net result of these effects is a shift of the marginal-revenue-product curve to the right (in general), which leads to the equilibrium point *B*. Generating points such as *A* and *B* at various levels of *w* we obtain the demand curve of labour. Apparently the demand for a variable factor is more elastic when several variable inputs are used in the production process. We conclude that the demand curves for inputs are negatively sloped, irrespective of the conditions of competition in the product market.

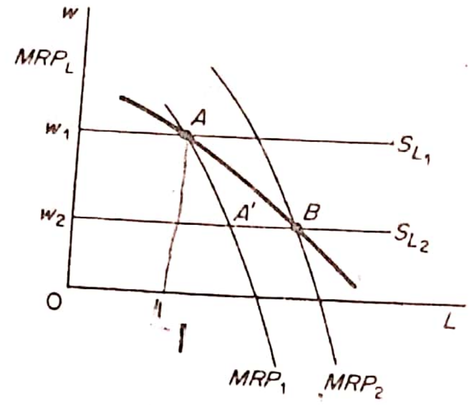


Figure 21.22

(c) The market demand for and supply of labour

The market demand for a factor is the summation of the demand curves of the individual monopolistic firms. In aggregating these curves, however, we must take into account their shift as the price of the factor falls: as *all* monopolistic firms expand their output, the market price falls. The individual demand curves and marginal revenue curves for the commodity produced shift to the left. Graphically the derivation of the market demand curve for labour is exactly the same as that in figure 21.13 (p. 447), except that the individual demand curves are based on the marginal revenue product of the factor (and *not* on the value of the marginal product, as in the case of a perfectly competitive product market, where P_x is given for all firms).

The market supply is not affected by the fact that firms have monopolistic power. Thus the market supply of labour is the summation of the supply curves of individuals, as derived earlier.

The market price of the factor is determined by the intersection of the market demand and the market supply. Thus the analysis does not change. However, there is an important difference: the market demand is based on the MRP_L and *not* on the VMP_L. This means that when the firms have monopolistic power the factor is paid its *MRP* which is smaller than the *VMP*. This effect has been called *monopolistic exploitation* by Joan Robinson.[1] It is shown in figure 21.23 for an individual firm, and in figure 21.24 for the labour market.

[1] Joan Robinson, *The Economics of Imperfect Competition* (Macmillan, 1933).

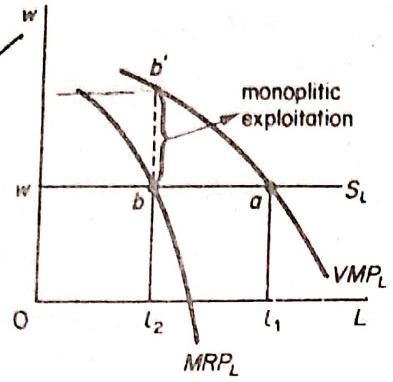


Figure 21.23

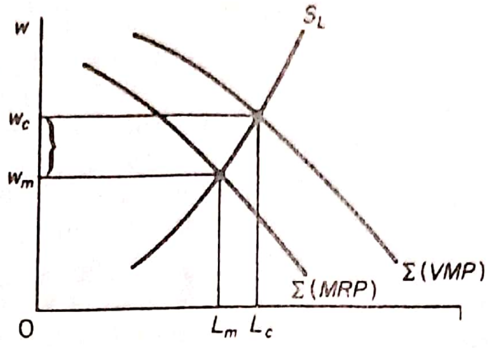


Figure 21.24

According to Joan Robinson a productive factor is exploited if it is paid a price less than the value of its marginal product (*VMP*). We saw that a profit maximiser will employ a factor until the point where an additional unit adds precisely the same amount to total cost and total revenue.

For a perfectly competitive firm

$$VMP_L = MRP_L$$

Because:

$$VMP_L = MPP_L \cdot P_x$$

and

$$MRP_L = MPP_L \cdot MR_x$$

But

$$P_x = MR_x$$

so that

$$VMP_L = MRP_L$$

Thus when the commodity market is perfectly competitive the profit maximising behaviour of firms leads to the payment of factor prices which are equal to the value of the marginal product (*VMP*) of the factors.

However, the condition of equilibrium of a monopolistic firm is

$$MRP_L = MPP_L \cdot MR_x = w$$

and $MR_x < P_x$, so that factors are paid less than the value of their marginal product. The difference bb' in figure 21.23 and $w_c w_m$ in figure 21.24 shows that profit-maximising behaviour of imperfectly competitive firms causes the factor price to be less than the value of its marginal product. Furthermore the level of employment is lower in industries which are not perfectly competitive $(L_c > L_m)$.

Joan Robinson's argument of 'exploitation' cannot be accepted on its face value. The fact that labour gets a lower wage in industries where competition is imperfect reflects the downward slope of the firms' demand curve, which is due to the brand loyalty of consumers. Product differentiation reflects consumers' desire for variety:

consumers want to be able to choose among substitute products. The consequence of this desire is a divergence between price and marginal revenue and a lower wage. The lower wage is thus the price that consumers must pay for having a variety of the same product, and cannot be considered as exploitation of labour by firms. Only if product differentiation is excessive, or is imposed on consumers by large corporations, one could accept the argument of labour exploitation in monopolistic markets.

Model B. The firm has monopolistic power in the commodity market and monopsonistic power in the factor market

(a) Equilibrium of a monopsonist who uses a single variable factor

In this case the demand for labour by the individual firm is the same as in Model A. That is, the demand for labour by a monopolistic firm is the MRP_L.

The supply of labour to the individual firm, however, is not perfectly elastic, because the firm is large. For simplicity we assume that the firm is a monopsonist (i.e. the only buyer) in the labour market. In this case the supply of labour has a positive slope: as the monopsonist expands the use of labour he must pay a higher wage (figure 21.25). The supply of labour shows the average expenditure or price that the monopsonist must pay at different levels of employment. Multiplying this price of the input by the level of employment we find the total expenditure of the monopsonist for the input. However, the relevant magnitude for the equilibrium of the monopsonist is the *marginal expenditure* of purchasing an additional unit of the variable factor.[1] The marginal expense is the change in the total expenditure (on the factor) arising from hiring an additional unit of the factor. Hiring an additional unit of input increases the total expenditure on the factor by more than the price of this unit because all previous units employed are paid the new higher price. Thus the *marginal expense curve* lies above the supply curve (or average expense curve).[2]

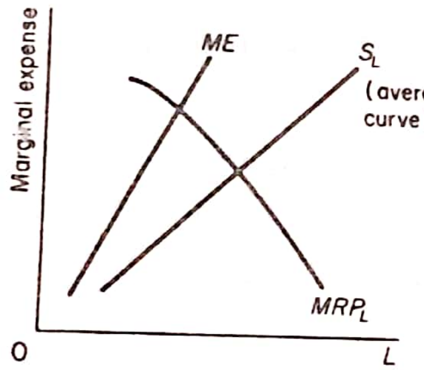


Figure 21.25

The *marginal expense* is the difference in the total expenditure at successively higher levels of employment of the factor. Table 21.3 illustrates the calculation of the marginal-expense-for-labour schedule. It is seen from table 21.3 that since the price per unit of input rises as employment increases, the marginal expense (ME) of the input is greater than its price at all levels of employment. The ME curve has a

[1] It is assumed that a single variable factor is used in the production process.

[2] This is the mirror image of the price-marginal revenue relation: the demand curve shows the price of X, and the MR_x lies below the demand curve at all levels of output.

Table 21.3 Total and marginal expense on labour

Units of labour	*Price of labour*	*Total expenditure on labour*	*Marginal expense on labour*
1	£4	£4	£—
2	5	10	6
3	6	18	8
4	7	28	10
5	8	40	12
6	9	54	14
7	10	70	16
8	11	88	18
9	12	108	20
10	13	130	22

positive slope and lies above and to the left of the supply of the input curve. This implies that the slope of the *ME* curve is greater than the slope of the supply curve, assuming linear relations.

Formal derivation of the ME *curve*

1. The input supply function is

$$w = f_1(L) \quad (1)$$

Its slope is dw/dL, and *ex hypothesi* $dw/dL > 0$. The total expense on the factor is

$$TE = w \cdot L$$

2. By definition the marginal expense is the change in the *TE* when *L* changes by one unit

$$ME = \frac{d(TE)}{dL} = w \cdot \frac{dL}{dL} + L \cdot \frac{dw}{dL}$$

or

$$ME = w + L \cdot \frac{dw}{dL} \quad (2)$$

Since

$$\frac{dw}{dL} > 0, \qquad L > 0 \text{ and } w > 0$$

it follows that *ME* is greater than *w* for any value of *L*.

3. The slope of the supply function is

$$\frac{dw}{dL} > 0$$

The slope of the *ME* curve is

$$\frac{d(ME)}{dL} = \frac{dw}{dL} + \left(\frac{dw}{dL} \cdot \frac{dL}{dL} + L \cdot \frac{d^2w}{dL^2}\right)$$

or

$$\frac{d(ME)}{dL} = 2 \cdot \frac{dw}{dL} + L \cdot \frac{d^2w}{dL^2}$$

Thus the *ME* has a steeper slope than the *w* (or S_L) function, assuming linear functions.

Q.E.D.

The firm is in equilibrium when it equates the marginal expenditure on the factor to its *MRP*. This is shown by point *e* in figure 21.26. The proof of the equilibrium is based on the definitions of the *ME* and MRP_L curves. At *e* the marginal expense (which is the marginal cost) of labour is equal to the marginal revenue product of labour. To the left of *e* a unit of labour adds more to the revenue than to the cost of the input; hence it pays the firm to hire additional units of labour. To the right of *e* an additional unit of the factor costs more than the revenue it brings to the firm; hence the profit is decreased. It follows that profit is maximised by employing that quantity of labour (l_e in figure 21.26) for which the *ME* is equal to the marginal revenue of the input.

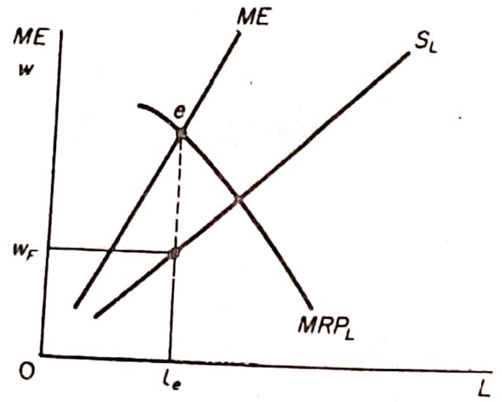


Figure 21.26

ME = MRP_L at eqm.

The wage rate that the firm will pay for the l_e units of labour is w_F, defined by the point on the supply curve corresponding to the equilibrium point *e*. To state this differently, w_F is the equilibrium wage corresponding to the equilibrium level of employment (l_e in figure 21.26).

When the firm has monopsonist power in the input market it pays to the factor a price which is less (not only than its *VMP*, but also less) than its *MRP*. This gives rise to *monopsonistic exploitation*, which is something in addition to monopolistic exploitation. We saw that monopolistic exploitation arises from the fact that the demand for the commodity is negatively sloped so that $MR_x < P_x$. The factor owners in this case are paid a price equal to the *MRP* of the factor, which is less than the factor's *VMP*. Monopsonistic exploitation arises from the monopsonistic power of firms, and is something in addition to the monopolistic exploitation. To illustrate this it is convenient to begin from the equilibrium wage rate when both the product and the factor markets are perfectly competitive. In figure 21.27 the *VMP* curve is the industry demand curve for labour[1] and the S_L curve is the market supply of labour (free from any union activity). Demand and supply intersect at point *A*, and the equilibrium wage rate is w_c: workers are paid the value of their marginal physical product (*VMP*).

Next assume that the commodity market is monopolistic, while the input market is perfectly competitive. The market demand for labour now is the *MRP* curve. It is the summation of shifting individual demand curves for labour. The labour market is in equilibrium at point *B* (figure 21.28). The difference between w_c and w_M is the 'monopolistic exploitation'. Each unit of labour receives its *MRP* which is less than the

[1] Recall that the market demand for labour is the sum of the shifting individual firms' demand curves for labour, as the price of the commodity falls when all firms increase their output in response to a decline in the wage rate.

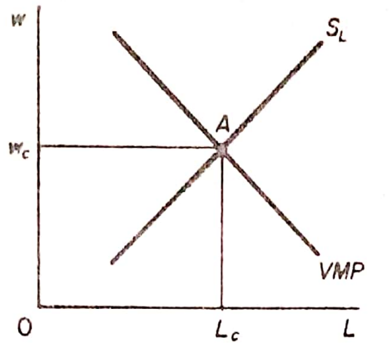


Figure 21.27 Perfectly competitive labour market

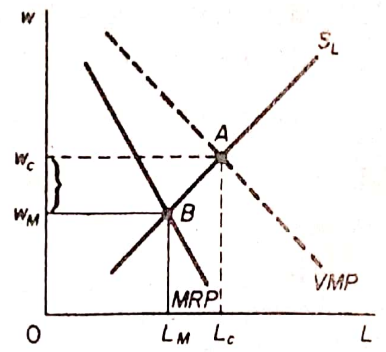


Figure 21.28 Monopolistic exploitation: $w_c - w_M$

VMP. Due to the monopoly power of firms, the price of the commodity will be higher and its demand lower. Hence the demand for labour will also be lower. Thus the result of 'monopolistic exploitation' is a lower wage rate and a lower level of employment.

Finally, assume that the firm is a monopsonist in the labour market and a monopolist in the product market. Its equilibrium is shown by point *C* in figure 21.29. The firm equates the *ME* of labour with its *MRP* (point *e* in figure 21.29). To maximise its profits the firm pays an even lower wage rate (w_s) and reduces further its employment (L_s). Monopsonistic exploitation is shown by the difference of the competitive wage and the monopsonistic wage ($w_c - w_s$). This can be split into two parts. The part $w_c w_M$ is due to the monopoly power of the firm; it would exist even if the firm were not a monopsonist in the labour market; hence this part is not uniquely attributable to monopsonistic elements. However, the part $w_s w_M$ is due to the monopsonistic power of the firm in the labour market. This power enables the firm to pay a wage rate lower than the *MRP* of labour.

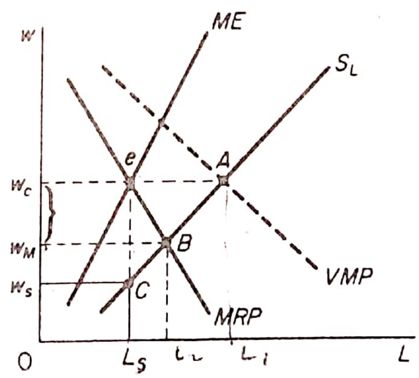


Figure 21.29

In summary.

(*a*) In perfectly competitive markets the factor is paid its *VMP*.

(*b*) If the firm has monopolistic power in the product market but no power in the input market, the factor is paid its $MRP < VMP$.

(*c*) If the firm has both monopolistic power in the product market and monopsonistic power in the input market the factor is paid a price which is even lower than its *MRP*. This is the basic characteristic of monopsonistic exploitation. The input price is determined from the S_L curve so that the factor does not get its MRP, which is its contribution to the total receipts of the firm.[1]

In the above analysis it was assumed that the market supply of labour is free from unionisation. This assumption will be relaxed in the model of bilateral monopoly, which will be examined in a subsequent section.

Let us now extend Model B to the case of a monopsonist who uses several variable factors.

(b) *Equilibrium of a monopsonist who uses several variable factors*

We have shown in Chapter 3 that if the input markets are perfectly competitive the firm minimises its cost by using the factor combination at which

$$\frac{MPP_L}{MPP_K} = \frac{w}{r}$$

or

$$\frac{MPP_L}{w} = \frac{MPP_K}{r}$$

If the factor markets are monopsonistic, changes in the amount of factors employed causes changes in the prices of factors. Thus w and r are not given. The monopsonist must look at the marginal expense of the factors. It can be shown that a monopsonist who uses several variable factors will use the input combination at which the ratio of the MPP to the ME is equal for all variable inputs. The least-cost combination is obtained when the marginal rate of technical substitution ($MRTS_{L,K}$) equals the marginal expense of input ratio. For the two-input case the equilibrium condition of the monopsonist may be stated as follows

$$MRTS_{L,K} = \frac{MPP_L}{MPP_K} = \frac{ME_L}{ME_K}$$

or

$$\frac{MPP_L}{ME_L} = \frac{MPP_K}{ME_K}$$

[1] The size of monopsonistic exploitation depends on the structure of the commodity market. For example in figure 21.30 we show the case of a firm which is a monopsonist in the labour market, but sells its product in a perfectly competitive market. Equilibrium is attained at point G,

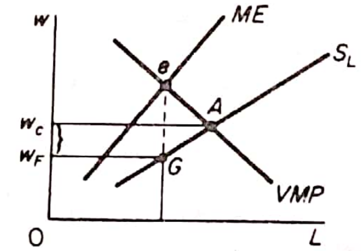


Figure 21.30

and monopsonistic exploitation is the total difference $w_c - w_F$, since there is no 'monopolistic exploitation' in this case. Thus, the above diagrammatic analysis can be applied to all types of market organisations (all degrees of imperfections).

University of Calcutta

M.Com – Semester II

CC-203 --- Operations Research- Module –II

Dr. S.P. RAY

PROBLEMS WITH *n* JOBS THROUGH TWO MACHINES AND FINALLY THROUGH THREE MACHINES

Practical Problem:1:

A readymade garment manufacturer has to process 7 items through two stages of production, namely cutting and sewing. The time taken for each of these at the different stages are given below in appropriate units:

Item		1	2	3	4	5	6	7
Cutting	Process time→	5	7	3	4	6	7	12
Sewing		2	6	7	5	9	5	8

(a) Find an order in which these items are to be processed through these stages, so as to minimize the total processing time.

(b) Suppose a third stage of production is added, namely, pressing and packing with the processing time as follows:

Items		1	2	3	4	5	6	7
Pressing and Packing	Processing time→	10	12	11	13	12	10	11

Find an order in which these seven items are to be processed so as to minimize the time taken to process all the items through all the stages.

Solution:

PART-A

In this problem, it is considered that the two stages of cutting and sewing are done by machine A and machine B.

The optimum sequence for these 7 items can be given as follows using the steps involved in Johnson's algorithm.

3	4	5	7	2	6	1

In order to find the total elapsed time and idle time for machine A and B,

Job	**Machine A**		**Machine B**		**Idle time**	
	In	**Out**	**In**	**Out**	**A**	**B**
3	0	3	3	10	-	3
4	3	7	10	15	-	-
5	7	13	15	24	-	-
7	13	25	25	33	-	1
2	25	32	33	39	-	-
6	32	39	39	44	-	-
1	39	44	44	**46**	-	-
					46-44=2	4

Total Elapsed time=46 hours

Idle time for machine A=2 hours

Idle time for machine B=4 hours

PART-B

In order to get the optimum sequence for including the third stage, namely, ***pressing and packing***, we use the optimum sequence for three machine problem by considering the stage – ***pressing and packing,*** for **machine C.**

We convert the problem into a two machine problem using the following steps:

$Min_i(A_i, C_i)$ **=(3,10), i=1,2,3……7**

and

$Max_i(B_i)$**=9**

Since $Min_i C_i$=10 $\geq$ $Max_i B_i$=9 is satisfied, we convert it into a two machine problem with machine G and H such that:

$$G_i = A_i + B_i \quad , i=1,2,3….7$$

$$H_i = B_i + C_i \quad , i=1,2,3….7$$

Items	1	2	3	4	5	6	7
Machine G	7	13	10	9	15	12	20
Machine H	12	18	18	18	21	15	19

We adopt Johnson's algorithm steps to get the optimum sequence.

1	4	3	6	2	5	7

In order to find the total elapsed time and idle time for machine A,B and C,

Job	Machine A		Machine B		Machine C		Idle time		
	In	***Out***	***In***	***Out***	***In***	***Out***	**A**	**B**	**C**
1	0	5	5	7	7	17	-	5	7
4	5	9	9	14	17	30	-	2	-
3	9	12	14	21	30	41	-	-	-
6	12	19	21	26	41	51	-	-	-
2	19	26	26	32	51	63	-	-	-
5	26	32	32	41	63	75	-	-	-
7	32	44	44	51	75	**86**	-	2	-
							86-44=42	86-51=35	-
							¤ 42	(35+5+2+2)= 44	7

Total elapsed time=86 hours

Idle time for machine A=42 hours; Idle time for machine B=44 hours;

Idle time for machine C=7 hours

PROBLEMS WITH *n* JOBS THROUGH THREE MACHINES

Practical Problem:2:

There are five jobs (namely 1,2,3,4 and 5), each of which must go through machines A, B and C in the order ABC. Processing Time (in hours) are given below:

Jobs	1	2	3	4	5
Machine A	5	7	6	9	5
Machine B	2	1	4	5	3
Machine C	3	7	5	6	7

Find the sequence that minimum the total elapsed time required to complete the jobs.

Solution

Here Min A_i = 5; B_i = 5 and C_i =3 since the condition of Min. $A_i \geq$ Max. B_i is satisfied ,the given problem can be converted into five jobs and two machines problem.

Jobs	$G_i = A_i + B_i$	$H_i = B_i + C_i$
1	7	5
2	8	8
3	10	9
4	14	11
5	8	10

The Optimal Sequence will be:

2	5	4	3	1

Total elapsed Time will be:

Jobs	Machine A		Machine B		Machine C	
	In	Out	In	Out	In	Out
2	0	7	7	8	8	15
5	7	12	12	15	15	22
4	12	21	21	26	26	32
3	21	27	27	31	32	37
1	27	32	32	34	37	**40**

Min. total elapsed time is 40 hours.

Idle time for Machine A is 8 hrs. (40-32)

Idle time for Machine B is [(7-0)+(21-15)+(27-26)+(32-31)+(40-34)]= 25 hours

(i.e 0-7, 8-12, 15-21, 26-27, 31-32 and 34-40)

Idle time for Machine C is[(8-0)+(26-22)]= 12 hours

(i.e 0-8, 22-26.)

PROBLEMS WITH *n* JOBS THROUGH TWO MACHINES

Practical Problem:3:

There are nine jobs, each of which must go through two machines P and Q in the order PQ, the processing times (in hours) are given below:

Machine	Job(s)								
	A	B	C	D	E	F	G	H	I
P	2	5	4	9	6	8	7	5	4
Q	6	8	7	4	3	9	3	8	11

Find the sequence that minimizes the total elapsed time T. Also calculate the total idle time for the machines in this period.

Solution

The minimum processing time on two machines is 2 which correspond to task A on machine P. This shows that task A will be preceding first. After assigning task A, we are left with 8 tasks on two machines

Machine	B	C	D	E	F	G	H	I
P	5	4	9	6	8	7	5	4
Q	8	7	4	3	9	3	8	11

Minimum processing time in this reduced problem is 3 which correspond to jobs E and G (both on machine Q). Now since the corresponding processing time of task E on machine P is less than the corresponding processing time of task G on machine Q therefore task E will be processed in the last and task G next to last. The situation will be dealt as

A							G	E

The problem now reduces to following 6 tasks on two machines with processing time as follows:

Machine	B	C	D	F	H	I
P	5	4	9	8	5	4
Q	8	7	4	9	8	11

Here since the minimum processing time is 4 which occurs for tasks C and I on machine P and task D on machine Q. Therefore, the task C which has less processing time on P will be processed first and then task I and task D will be placed at the last i.e., 7^{th} sequence cell.

The sequence will appear as follows:

A	C	I				D	E	G

The problem now reduces to the following 3 tasks on two machines

Machine	B	F	H
P	5	8	5
Q	8	9	8

In this reduced table the minimum processing time is 5 which occurs for tasks B and H both on machine P. Now since the corresponding time of tasks B and H on machine Q are same i.e. 8. Tasks B or H may be placed arbitrarily in the 4^{th} and 5^{th} sequence cells. The remaining task F can then be placed in the 6^{th} sequence cell. Thus the optimal sequences are represented as

A	I	C	B	H	F	D	E	G

or

A	1	C	H	B	F	D	E	G

Further, it is also possible to calculate the minimum elapsed time corresponding to the optimal sequencing $A \rightarrow I \rightarrow C \rightarrow B \rightarrow H \rightarrow F \rightarrow D \rightarrow E \rightarrow G$.

Job Sequence	Machine A		Machine B	
	Time In	Time Out	Time In	Time Out
A	0	2	2	8
I	2	6	8	19
C	6	10	19	26
B	10	15	26	34
H	15	20	34	42
F	20	28	42	51

D	28	37	51	55
E	37	43	55	58
G	43	50	58	**61**

Hence the total elapsed time for this proposed sequence staring from job A to completion of job G is **61 hours** .During this time machine P remains idle for **11 hours** (from 50 hours to 61 hours)and the machine Q remains idle for **2 hours** only (from 0 hour to 2 hour).

PART- II [MCQ Questions]

1. In Sequencing Algorithm:

(a) The selection of an appropriate order for a series of jobs is to be done on a finite service facilities

(b) All the jobs must be processed on a first come –first serve basis

(c) A service facility can process more than one job at a time

(d) All the service facilities are not of different type

Ans: 1(a)

2.The general assumption which is not correct in solving a sequencing problem is that

(a) the time taken by different jobs in moving from one machine to another is negligible.

(b) the processing time in various machines are independent of the order in which different jobs are processed on them.

© a job once started on a machine would be performed to the point of completion uninterrupted.

(d)a machine can process more than one job at a given point of time.

Ans: 2(d)

3.In "n" jobs and two machines (say A and B) sequencing problems in which the order of processing is AB:

(a) job having minimum time on machine B is processed first.

(b) job having minimum time on machine A is processed in the last.

© job having minimum time on machine B is processed in the last.

(d)job having maximum time on machine B is processed in the last.

Ans: 3(c)

4. Five jobs are to be processed on three machines A,B and C in the order ABC. The timing of the jobs are known to be : (30,40,70) , (80,50,90), (70,10,50), (50,20,60), and (40,30,100)

The optimum sequence would be:

(a) $J_1 \to J_4 \to J_5 \to J_2 \to J_3$

(b) $J_1 \to J_4 \to J_2 \to J_5 \to J_3$

(c) $J_4 \to J_1 \to J_3 \to J_2 \to J_5$

(d) $J_4 \to J_1 \to J_5 \to J_2 \to J_3$

Ans:8(d)

5. If A_i, B_i and C_i denote the processing times of i th job on three machines A,B and C respectively, then a "n" –job three machine problem can be reduced to an n –job and two machine problem, provided that:

(a) $\min A_i \geq \max B_i\, and / or \min C_i \leq \max B_i$

(b) $\min A_i \geq \max B_i\, and / or \min C_i \geq \max B_i$

© $\min A_i \leq \max B_i\, and / or \min C_i \geq \max B_i$

(d) $\min A_i \leq \max B_i\, and / or \min C_i \leq \max B_i$

Ans:5(b)

Study material prepared by Dr S.P.Ray

For further practice, please refer

1. Operations Research- Theory and Applications – J.K.Sharma

2. Operations Research- Problems and Solutions – V.K.Kapoor

Lesson II

(Prof. S. Bhattacharyya)

Sl. No.	**GST; Some Statements**	
1	**GST is an Indirect Tax** i.e., the person depositing the sum with the revenue authority can legally shift the burden on his/her customer. ***Exception***:GST does not behave like an Indirect Tax in case of (**Composition Levy**)	
2	**It arises on "Supply" of Goods or Services or both**	
3	**Supply involves two persons – Supplier and Recipient of Supply.** • **Any person other than Supplier and Recipient is third person**	
4	**To attract GST at least one of the persons (Supplier/Recipient) in the supply function must be a person in business**	
5	**Only a Registered Supplier can collect Tax on outward supply and only a Registered Supplier can claim Input Tax Credit**	
6	Under GST Tax liability is that of the Taxable Person • A registered Supplier is a Taxable Person (called ***Forward Charge***) • In case of ***Reverse Charge*** Recipient of Supply is the Taxable Person GST revenue is deposited with the revenue authority. Deposit of tax with revenue authority is made in case of: • Forward Charge ***by the Supplier*** • Reverse Charge ***by the Recipient of supply***	
7	**GST is a Value Addition based Tax** A Value Addition based tax is one that • Removes cascading effect on price and • Reduces Tax on Output by Tax paid on Input(known as**Input Tax Credit**)	
8	**GST is a Destination based Tax (**also called **Consumption based tax)** • Destination: **Place of final consumption of the Goods or Services or both** Under GST :- • *Nature of supply depends on*: Location of supplier and Place of supply	
9	Under GST :- Tax liability is determined from: Value of Supply and Rate of tax Rate of Taxdepends on: [Time of Supply, Type of Supply(Mixed/Composite) and Nature of supply(Intra-state/Inter State)]	

GST is an Indirect Tax

- A Direct Tax directly affects the income/wealth of the taxable person. The burden of such tax cannot be shifted on another person.
- The burden of tax can legally be shifted on another person if the tax concerned is an indirect tax. Burden of such tax is finally borne by the consumer.
  In other words, under an indirect tax law the payer of tax(other than a consumer) can shift the burden of tax on another person.

GST is an Indirect Tax because the burden of GST can legally be shifted on another person

It is concerned with "Supply" of Goods or Services or both

SUPPLY

- ***When recognised?At the*****Time of Supply ………Imposition(Point oftaxation)**
- ***Where completed?At the*****Place of Supply…….Completion(Nature of supply)**
- **Occurred*at what value?At the*Value of Supply….Valuation(Assessable Value)**

- **Time of Supply determines**
- **Point of Taxation (i.e., levy)**
- **Rate of Tax applicable**
- **Time of payment of Tax**

- **Place of supply determines**
- **category of supply (Intra-state/Inter state) [read with "Location of Supplier"] to determine the type of GST[(CGST + SGST)/ IGST]**
- **the state to enjoy the GST revenue**

- **Value of Supply determines**
  - **the Assessable Value**

Problem No. 2(**Revised)**

Q. 2:Subir of West Bengal imported some goods, for the first time, and supplied the same to Amrik of Punjab for Rs. 60,000. Subir and Amrik are related persons. Price of similar goods available in the market is Rs. 75,000. Applicable rate of GST on such supply:- CGST 6%, SGST 6% and IGST 12%.
Subsequently Amrik supplied goods to Arora of Punjab for Rs. 90,000. Applicable rate of GST on such supply:- CGST 9%, SGST 9% and IGST 18%.
Calculate the GST liability of Subir and Amrik.
Solution:

Taxable Person	**Subir**
Supply	**Inter state**
Value of supply	.75,000**
Tax on outward supply (IGST)	. 9,000
Input Tax Credit	NIL
GST payable (IGST)	. 9,000

**Reason: Related person. Open Market Value not available. Hence price of similar goods considered.

Taxable Person	**Amrik**	
Supply	**Intra-state**	
Value of supply	Rs.90,000	
	CGST	SGST
Tax on outward supply :	Rs. 8,100	Rs. 8,100
Input Tax Credit(IGST)	Rs. 8,100	Rs. 900(9,000 – 8,100)
GST payable	NIL	Rs. 7,200

Tax Liability under some special cases

1. **Mixed Supply:** A mixed supply is two or more independent products or services which are offered together as a bundle but can also be sold separately. *The mixed supply is taxed, at the rate, applicable to the item or service with the highest GST rate in the supply concerned.*
   Example: On the occasion of puja Mirror, comb and hair band are supplied in a single pack at a consolidated price.

- Each of the items in the packet can be supplied separately and is not dependent on each other. The supply is not naturally bundled. Hence it is a mixed supply.

2. **Composite Supply:** A composite supply is two or more goods or services that are only sold as a set and cannot be sold individually. Every composite supply has a principal supply, which is the main product or service that the buyer primarily wants. The rest of the supply is made up of supporting elements that add value to the principal supply.
   A composite supply is taxed at the GST rate of the principal supply.

   Example: A charger has been supplied by the a supplier along with the mobile phone.

- Supply of charger along with mobile phone is in conjunction with each other as it is required by the user of the mobile phone. So this supply is *naturally bundled*. Hence it is a composite supply.

3. **Free Samples and gifts**

- Government has clarified that samples supplied free of cost, without any consideration, do not qualify as supply under GST. Example: Physician's sample.

4. **Buy one get one free offer**

   Example: Buy one Suthol liquid and get one suthol gel tube free / Get one tooth brush free along with the purchase of one "Neem" tooth paste.
   This type of offer is a case of two or more individual supplies where a single price I being charged for the entire supply. In other words it is a case of supplying two goods for a single price.

- Government has clarified that the taxability of such supply will be dependent upon as to whether the supply is a composite supply or a mixed supply and rate tax will be determined accordingly.

PLACE OF SUPPLY

Under an ideal taxation system, tax shall only form part of cost when the goods or services reach the final consumer and businesses should be allowed credit of whatever taxes they have paid unless the goods /services supplied by them are exempt from levy of tax or not liable to be taxed.

GST is a consumption-based/ destination- based Tax Or **Under GST :- Sharing of Tax Revenue = f(Place of Supply)**

Meaning of 'Destination'

Destination based taxation is a system wherein revenue from tax relating to goods/services accrues to the jurisdiction where there being ultimately consumed. It is also called consumption tax.**)**Thus for the purpose of this Act 'Destination' is the place of consumption

IMPORTANCE of "Place of Supply"

Place of supply is an important ingredient so that the type of tax that is to be applied may be correctly determined. It should not be understood in the lay man's language. Rather, it is a phrase having legal meaning and should be understood in that sense only.

It determines-

- Whether a supply is an intra-state/inter-statesupply.
- State to share the tax revenue.

Whether a supply is an intra-state/inter-state supply depends on Location of Supplier and Place of Supply

Intra-state supply: Location of Supplier and Place of Supply are in the same state

Inter-state supply: Location of Supplier and Place of Supply are in two different states

Location of supplier	***Place of supply***	***Nature of supply***	***GST payable***
In the same state		**Intra-state**	**CGST-SGST/UGST**
In two different states		**Inter state**	**IGST**

Principle behind the provision

GST is a destination based consumption tax. But there is no provision that declares this fact. This missing declaration is more than adequately supplied by the principle being embodied in the provisions of 'Place of Supply'.The basic principle behind provisions relating toplace of supply is that GST is a destination based tax. Thus, tax is finally payable where goods and/services are consumed.

Location of Supplier of Goods
It is the geographical point(e.g., premises of supplier/site of supplier) where the supplier is situated with goods(in its control ready to be supplied). Location of supplieris usually the place from where a supply is made.***A place mentioned as a principal place of business on the GST registration Certificate, may be taken as location of supplier***.

Location of Supplier of Services ----- defined in Sec 2(15) of the IGST Act

Simplified

A. *When supply is made from one establishment*
   i) Place mentioned in the registration certificate --- location of such place of business
   ii) A fixed establishment [other than (i)] ---- location of such fixed establishment

B. *When supply is made from more than one establishment*
   Whether from A(i)/(ii)…location of the establishmentmost directly concerned with the provision of the supply

C. In the absence of A/B …….. Usual place of residence of the supplier

Tabular Presentation

A.	*When **supply i**s made **from one establishment***	**Location of Supplier of Services**
	(i)Place mentioned in the registration certificate	**Place mentioned in the registration certificate**
	(ii) A fixed establishment [other than (i)]	location of ***such fixed establishment***
B.	*When **supply** is made **from more than one establishments*** **whether from A(i)/(ii)**	location of the ***establishment most directly concerned*** with the provision of the supply
C.	In the absence of A/B	Usual ***place of residence*** of the supplier