Department of Commerce

University of Calcutta

Study Material

Cum

Lecture Notes

Only for the Students of M.Com. (Semester II)-2020

University of Calcutta

(Internal Circulation)

Dear Students,

Hope you, your parents and other family members are safe and secured. We are going through a world-wide crisis that seriously affects not only the normal life and economy but also the teaching-learning process of our University and our department is not an exception.

As the lock-down is continuing and it is not possible to reach you face to face class room teaching. Keeping in mind the present situation, our esteemed teachers are trying their level best to reach you through providing study material cum lecture notes of different subjects. This material is not an exhaustive one though it is an indicative so that you can understand different topics of different subjects. We believe that it is not the alternative of direct teaching learning.

It is a gentle request you to circulate this material only to your friends those who are studying in Semester II (2020).

Stay safe and stay home.

Best wishes.

For

Semester-II

[Additional Materials]

SERIES-III

Transportation Problems

❑ Initial Basic Feasible Solution (IBFS):

Question No. 1:

Obtain an initial basic feasible solution to the following transportation problem by the following methods:

(i) North-West Corner Method (NWCM);
(ii) Least Cost Method (LCM);
(iii) Vogel's Approximation Method (VAM).

Warehouses	**Stores**				Availability
	I	**II**	**III**	**IV**	
A	7	3	5	5	34
B	5	5	7	6	15
C	8	6	6	5	12
D	6	1	6	4	19
Demand	21	25	17	17	**80**

Solution:

➢ (i) Solution by NWCM:

(a) First check whether the problem is balanced or not i.e. whether total demand is equal to total supply i.e.

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$$

Where S = Supply, D = Demand, m = No. of sources n = No. of destinations

If the problem is not balanced, then balance it.

(b) Now, obtain IBFS by NWCM as follows:

To / from	Stores I	II	III	IV	Availability
A	(21) 7	(13) 3	5	5	~~34~~ ~~13~~ 0
B	5	(12) 5	(3) 7	6	~~15~~ ~~3~~ 0
C	8	6	(12) 6	5	~~12~~ 0
D	6	1	(2) 6	(17) 4	~~19~~ 17
Demand	~~21~~ 0	~~25~~ ~~12~~ 0	~~17~~ ~~14~~ ~~2~~ 0	17	80

Therefore, the initial solution is

= Rs. (21 × 7 + 13 × 3 + 12 × 5 + 3 × 7 + 12 × 6 + 2 × 6 + 17× 4) = Rs 419

➢ (ii) Solution by LCM:

(a) First check whether the problem is balanced or not i.e. whether total demand is equal to total supply.

(b) Now, obtain IBFS by LCM as follows:

(ii) From / To Warehouse	I	II	III	IV	Availability
A	7 (6) 7	2 (6) 3	3 (17) 5	6 (5) 5	~~34~~ ~~28~~ ~~10~~ ~~11~~ ~~6~~
B	4 (15) 5	5	7	6	~~15~~ 0
C	8	6	6	5 (12) 5	~~12~~ 0
D	6	1 (19) 1	6	4	~~19~~ 0
Demand	~~21~~ ~~6~~ 0	~~25~~ ~~6~~ 0	~~17~~ 0	~~17~~ ~~5~~ 0	80

Therefore, the initial transportation cost is

= Rs. (6 × 7 + 6 × 3 + 17 × 5 + 5 × 5 + 15 × 5 + 12 × 5 + 19× 1) = Rs 324

➢ (iii) Solution by VAM:

(a) First check whether the problem is balanced or not i.e. whether total demand is equal to total supply.

(b) Now, obtain IBFS by VAM as follows:

(iii) From / To	I	II	III	IV	S	Row Penalty 1 2 3 4 5
A	7 (6) 7	2 (6) 3	4 (17) 5	6 (5) 5	~~34~~ ~~28~~ ~~11~~ ~~6~~ 0	2 2 0 0 2
B	3 (15) 5	5	7	6	~~15~~ 0	0 0 1 – –
C	8	6	6	5 (12) 5	~~12~~ 0	1 1 1 1 (3)
D	6	1 (19) 1	6	4	~~19~~ 0	(3) – – – –
Demand	~~21~~ ~~6~~ 0	~~25~~ ~~6~~ 0	~~17~~ 0	~~17~~ ~~5~~ 0	80	
Column Penalty	1	2	1	1		
	2	(2)	1	0		
	(2)	–	1	0		
	1	–	(1)	0		
	1	–	–	0		

Therefore, the initial transportation cost is

= Rs. (6 × 7 + 6 × 3 + 17 × 5 + 5 × 5 + 15 × 5 + 12 × 5 + 19× 1) = Rs 324

Note:

Students should note that VAM gives the best initial basic feasible solution. As we proceed from NCWM to VAM, we would observe that solution given by NWCM is not the best solution. In this case, the cost would be higher than the initial solution obtained through LCM and VAM. In present problem, the solutions of LCM and VAM show the same results, but in many cases, you will find that VAM gives the least cost solution i.e. the best solution among the three methods of obtaining IBFS. If no method is specified for initial solution in examination, then VAM is preferred over all other methods and students should solve the question accordingly.

❑ Maximisation Type Problem- Unbalanced One, Use of MODI:

Question No. 2:

Following is the profit matrix based on 4 factories and 3 sales depots of XYZ. Ltd.:

Factories	Sales Depots			Availability (No.)
	S_1	S_2	S_3	
F_1	6	6	1	10
F_2	-2	-2	-4	150
F_3	3	2	2	50
F_4	8	5	3	100
Requirement (No.)	80	120	150	

Determine the most profitable distribution schedule and the corresponding profit, assuming no profit in case of surplus production.

Solution:

➢ Initial Solution by VAM:

(a) The given transportation problem is an unbalanced one and it is a maximisation type problem.

The total requirement = 80 +120 + 150 = 350

The total availability = 10 + 150 + 50 + 100 = 310

Therefore, S ≠ D

Hence, Dummy Factory with 40 units of supply is created and is shown as follows:

Factories	Sales Depots			Availability (No.)
	S_1	S_2	S_3	
F_1	6	6	1	10
F_2	-2	-2	-4	150
F_3	3	2	2	50
F_4	8	5	3	100

Dummy	0	0	0	40
Requirement (No.)	80	120	150	**350**

(b) We shall now convert the above profit matrix into a loss matrix by subtracting all the elements from the highest value in the table i.e. 8 (being the highest). The loss matrix is shown below:

Factories	**Sales Depots**			Availability (No.)
	S_1	S_2	S_3	
F_1	2	2	7	10
F_2	10	10	12	150
F_3	5	6	6	50
F_4	0	3	5	100
Dummy	8	8	8	40
Requirement (No.)	80	120	150	**350**

Note: Calculation is done as follows:

$F_1S_1 = 8 - 6 = 2$, $F_2S_1 = 8 - (-2) = 10$, $DummyS_1 = 8 - 0 = 8$, $F_3S_3 = 8 - 2 = 6$ etc.

(c) Then we apply VAM for finding out IBFS: [The Rule is same like the previous Solution 1 (iii)]

IBFS

	S_1	S_2	S_3	Av.	Row Penalty				
F_1	2	(10) 2	7	~~10~~ 0	0	5	-	-	-
F_2	10	(40) 10	(110) 12	~~150~~ ~~110~~ 0	0	2	2	2	2
F_3	5	(50) 6	6	~~50~~ 0	1	0	0	0	-
F_4	(80) 0	(20) 3	5	~~100~~ ~~20~~ 0	3	2	2	-	-
Dum.	8	8	(40) 8	~~40~~ 0	0	0	0	0	0
Req.	~~80~~ 0	~~120~~ ~~110~~ ~~90~~ ~~40~~ 0	~~150~~ ~~110~~ 0	350					
Column Penalty	2	1	1						
	-	1	1						
	-	3	1						
	-	2	2						
	-	2	4						

(d) Next the initial solution obtained by VAM is tested for optimality using Modified Distribution (MODI) approach.

(e) The IBFS is non-degenerate as the total number of independent allocations is 7 which is equal to the condition $(\mathbf{m + n - 1}) = 5 + 3 - 1 = 7$ allocations.

(f) Now let us introduce column U_i to indicate row values and row V_j to indicate column values, where, $i = 1, 2, ..., 5$ and $j = 1, 2, 3$ such that $\Delta_{ij} = C_{ij} - (U_i + V_j)$ for all **unallocated cells**. Therefore, for each unoccupied cell, the opportunity cost is determined using the formula:

$\Delta_{ij} = C_{ij} - (U_i + V_j)$

(g) The unit transportation cost of the 7 occupied cells can be calculated as follows:

We assume $V_2 = 0$ (You can assume any U_i or V_j as "0")

For occupied cell,

$C_{12} = U_1 + V_2 = 2 \Rightarrow U_1 = 2$

$C_{22} = U_2 + V_2 = 10 \Rightarrow U_2 = 10$

$C_{23} = U_2 + V_3 = 12 \Rightarrow V_3 = 2$

$C_{32} = U_3 + V_2 = 6 \Rightarrow U_3 = 6$

$C_{41} = U_4 + V_1 = 0 \Rightarrow V_1 = - 3$

$C_{42} = U_4 + V_2 = 3 \Rightarrow U_4 = 3$

$C_{53} = U_5 + V_3 = 8 \Rightarrow U_5 = 6$

The values of Δ_{ij} for unoccupied cells are calculated accordingly.

	S_1	S_2	S_3	u_i
F_1	$\boxed{3}$ 2	10 2	$\boxed{3}$ 7	$u_1 = 2$
F_2	$\boxed{3}$ 10	$+\theta$ 40 10	$-\theta$ 110 12	$u_2 = 10$
F_3	$\boxed{2}$ 5	50 $-\theta$ 6	$\boxed{-2}$ $+\theta$ 6	$u_3 = 6$
F_4	80 0	20 3	$\boxed{0}$ 5	$u_4 = 3$
Dummy	$\boxed{5}$ 8	$\boxed{2}$ 8	40 8	$u_5 = 6$
V_j	$V_1 = -3$	$V_2 = 0$ (assume)	$V_3 = 2$	

(h) Since, one of the Δ_{ij} (i.e. Δ_{33}) is negative (= - 2), the above initial solution is not optimal, that means there is a scope of improvement. Now the negative delta value cell is made an allocated cell by transferring the minimum allocation as follows based on creation of close loop considering 4 cells such as F_2S_2, F_2S_3, F_3S_2 and F_3S_3:

Therefore, θ_{Max} = Min (50, 110) = 50 units to be transferred in F_3S_3.

The solution is shown below along with calculation of revised Δ_{ij} values for each unoccupied cell.

Let us assume now $U_2 = 0$ (You can assume any U_i or V_j as "0")

	S_1	S_2	S_3	u_i
F_1	$\boxed{3}$ 2	10 2	$\boxed{3}$ 7	$u_1 = -8$
F_2	$\boxed{3}$ 10	90 10	60 12	$u_2 = 0$
F_3	$\boxed{4}$ 5	$\boxed{2}$ 6	50 6	$u_3 = -6$
F_4	80 0	20 3	$\boxed{0}$ 5	$u_4 = -7$
Dumm	$\boxed{5}$ 8	$\boxed{2}$ 8	40 8	$u_5 = -4$
	$V_1 = 7$	$V_2 = 10$	$V_3 = 12$	

(i) Since all Δ_{ij} values are either positive or zero, the above solution is now optimal. The distribution schedule along with the profit is given below:

Factory	Sales Depot	Units	Profit per unit (₹)	Total Profit (₹)
F_1	S_2	10	6	60
F_2	S_2	90	-2	-180
F_2	S_3	60	-4	-240
F_3	S_3	50	2	100
F_4	S_1	80	8	640
F_4	S_2	20	5	100
			Total	480

The above questions and solutions are only explanatory and showing the type of questions. Students are also requested to practice a good number of different types of practical questions based on the above-mentioned topics from the text books already referred in class.

Assignment Problems

Methods of Solving Assignment Problems:

1. Complete Enumeration Method,
2. Transportation Method,
3. Simplex Method,
4. Hungarian Assignment Method (HAM).

Question No. 1: Balanced Assignment Problem

A particular department has 5 jobs and 5 subordinates as shown below. The number of hours each man would take to perform each job is shown as follows:

Subordinates	Jobs				
	1	**2**	**3**	**4**	**5**
A	3	5	10	15	8
B	4	7	15	18	8
C	8	12	20	20	12
D	5	5	8	10	6
E	10	10	15	25	10

You are required to assign the jobs among the subordinates in such a way that would minimise the total hours worked.

Solution:

(i) Solution by Hungarian Assignment Method (HAM):

The given problem is a balanced minimisation type assignment problem. Let us apply the assignment algorithm:

Step 1: Row Operation

Subtracting the smallest element of each row from all the elements of that row, we get the following reduced matrix (Here 3 is the smallest element in 1^{st} row, 4 is the smallest element in 2^{nd} row etc. Although in this question all smallest elements of the rows fall in the 1^{st} column, so there arises zero in 1^{st} column. This may not be true in all cases):

Reduced Matrix (Row-wise)

Subordinates	Jobs				
	1	**2**	**3**	**4**	**5**
A	0	2	7	12	5
B	0	3	11	14	4
C	0	4	12	12	4
D	0	0	3	5	1

E	0	0	5	15	0

Step 2: Column Operation (if required)

Subtracting the smallest element in each column from the all elements of that column of the reduced matrix, we get the following:

Reduced Matrix (Column-wise)

	1	2	3	4	5	
A	0	2	4	7	5	
B	0	3	8	9	4	
C	0	4	9	7	4	
D	0	0	0	0	1	L2
E	0	0	2	10	0	L3
	L1					

Since the numbers of lines covering all zeros are less than the number of rows/ columns, the solution is not optimal.

In order to improve he solution, we subtract the smallest uncovered element (i.e. 2) from all uncovered elements and add it (i.e. 2) to the elements lying on the intersection of two lines. We get the following matrix:

1	2	3	4	5	
0	0	2	5	3	L4
0	1	6	7	2	
0	2	7	5	2	
2	0	0	0	1	L2
2	0	2	10	0	L3
L1					

The solution is not optimal since the minimum number of lines covering all zeros is not equal to 5. Let us take smallest element (i.e. 1) from the uncovered cell and perform the same procedure as stated above. We get the following matrix:

	1	2	3	4	5	
A	1	0	2	5	3	
B	0	0	5	6	7	
C	0	1	6	4	1	
D	3	0	0	0	1	L1
E	3	0	2	10	0	L4
	L3	L2				

Again the solution is not optimal. Therefore, it is improved following the above-mentioned procedure. We get the following matrix (Again 1 is selected as the smallest element from the uncovered cells):

	1	2	3	4	5	
A	1	0	1	4	2	
B	0	0	4	5	0	
C	0	1	5	3	0	
D	4	1	0	0	1	L4
E	4	1	2	10	0	
	L2	L3			L1	

The above solution is not optimal. Again we select the smallest element from all uncovered cells (i.e. 1 is selected as the smallest uncovered element based on the above matrix) and perform the same procedure i.e. we subtract the smallest uncovered element (here 1) from all uncovered elements and add it (i.e. 1) to the elements lying on the intersection of two lines. We get the following matrix:

	1	2	3	4	5	
A	1	$\cancel{0}$	$\boxed{0}$ 2	3	2	L2
B	$\cancel{0}$	$\boxed{0}$ 3	3	4	$\cancel{0}$	
C	$\boxed{0}$ 4	1	4	2	$\cancel{0}$	
D	5	2	$\cancel{0}$	$\boxed{0}$ 1	2	L4
E	4	1	1	9	$\boxed{0}$	
	L1	L5			L3	

The above solution is optimal. Therefore on the basis of above optimal solution, the assignment of jobs to the subordinates will be as follows:

Subordinates	Jobs	Hours
A	3	10
B	2	7
C	1	8
D	4	10
E	5	10
Total		45 Hours

Question No. 2: Unbalanced Assignment Problem

In the modification of a plant layout of a factory, four machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are 5 vacant places J, K, L, M and N available. Because of limited space, M_2 cannot be placed at L and M_3 cannot be placed at J. The cost of placing machine *i* at place *j* (in Rupees) are show below:

Machines	Places				
	J	**K**	**L**	**M**	**N**
M_1	18	22	30	20	22
M_2	24	18	--	20	18
M_3	--	22	28	22	14
M_4	28	16	24	14	16

You are required to determine optimal assignment schedule in such a manner that the total costs are kept at a minimum.

Solution:

The given problem is unbalanced one and so we add one dummy machine with zero (0) cost. Also assign a high cost M (*it has no relation with the place M*) to the pair (M_2L) and (M_3J). The cost matrix is shown below:

Machines	Places				
	J	**K**	**L**	**M**	**N**
M_1	18	22	30	20	22
M_2	24	18	M	20	18
M_3	M	22	28	22	14
M_4	28	16	24	14	16
M_5 (Dummy)	0	0	0	0	0

Now, applying Hungarian Assignment Method (HAM), the optimal solution can be arrived as follows:

Step 1

Subtract the minimum element of each row from each element of that row-

	J	K	L	M	N
M_1	0	4	12	2	4
M_2	6	0	M	2	0
M_3	M	8	14	8	0
M_4	14	2	10	0	2
M_5 (Dummy)	0	0	0	0	0

Step 2

Subtract the minimum element of each column from each element of that column-

	J	K	L	M	N
M_1	0	4	12	2	4
M_2	6	0	M	2	0
M_3	M	8	14	8	0
M_4	14	2	10	0	2
M_5 (Dummy)	0	0	0	0	0

Step 3

Draw lines to connect the zeros as under-

	J	K	L	M	N
M_1	$\boxed{0}$	4	12	2	4
M_2	6	$\boxed{0}$	M	2	0
M_3	M	8	14	8	$\boxed{0}$
M_4	14	2	10	$\boxed{0}$	2
M_5 (Dummy)	0	0	$\boxed{0}$	0	0

There are five lines which are equal to the order of the matrix. Hence the solution is optimal. We may proceed to make the assignment as follows:

	J	K	L	M	N
M_1	$\boxed{0}$	4	12	2	4
M_2	6	$\boxed{0}$	M	2	$\cancel{0}$
M_3	M	8	14	8	$\boxed{0}$
M_4	14	2	10	$\boxed{0}$	2
M_5 (Dummy)	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

The following is the assignment which keeps the total cost at minimum:

Machines	Location	Costs (₹)
M_1	J	18
M_2	K	18
M_3	N	14
M_4	M	14
M_5 (Dummy)	L	0
	Total	64