UNIVERSITY OF CALCUTTA

Notification No. CSR/ 77 /18

It is notified for information of all concerned that the Syndicate in its meeting held on 13.07.2018 (vide Item No.11) approved the Syllabus of Two-Year (Four-Semester) M.Sc. Course of Study in Pure Mathematics under CBCS in the Post-Graduate Departments of the University and in the affiliated Colleges offering Post-Graduate Courses under this University, as laid down in the accompanying pamphlet.

The above shall be effective from the academic session 2018-2019.

17/08/18

(Debabrata Manna)
Deputy Registrar (Acting)

SENATE HOUSE
KOLKATA-700073
The 17th August, 2018

Department of Pure Mathematics

Course Structure for M.Sc. (w.e.f. July, 2018)

Semester-wise distribution of Courses (Under CBCS System)

Semester	Course ID	Group	Name of the Courses	Page Number	Full Marks	Credit Point	Classes per week
I	PM1/01	Gr.-A	Group Theory	4	25	4	5 hr
		Gr.-B	Ring Theory	5	25		
	PM1/02		Real Analysis -I	6	50	4	5 hr
	PM1/03	Gr.-A	Complex Analysis -I	8	25	4	5 hr
		Gr.-B	Ordinary Differential Equation	9	25		
	PM1/04	Gr.-A	General Topology -I	10	30	4	5 hr
		Gr.-B	Differential Geometry of Curves & Surfaces	11	20		
	PM1/05	Gr.-A	Discrete Mathematics -I	12	30	4	5 hr
		Gr.-B	Multivariate Calculus	13	20		
			Total		**250**	**20**	**25 hr**
II	PM2/06		Linear Algebra	14	50	4	5 hr
	PM2/07	Gr.-A	Real Analysis -II	16	25	4	5 hr
		Gr.-B	Complex Analysis -II	18	25		
	PM2/08		General Topology -II	19	50	4	5 hr
	PM2/09		Functional Analysis	21	50	4	5 hr
	PM2/10	Gr.-A	Discrete Mathematics - II	23	20	4	5 hr
		Gr.-B	Theory of Manifold	24	30		
			Total		**250**	**20**	**25 hr**
III	PM3/11	Gr.-A	Field Extension	26	25	4	5 hr
		Gr.-B	Algebraic Topology - I	27	25		
	PM3/E1/101-111		Elective -I	2	50	4	5 hr
	PM3/E2/201-211		Elective -II	3	50	4	5 hr
	CBCC - A		Choice Based Credit Course - A		50	4	5 hr
	CBCC - B		Choice Based Credit Course - B		50	4	5 hr
			Total		**250**	**20**	**25 hr**
IV	PM4/12	Gr.-A	Algebraic Topology - II	28	30	4	5 hr
		Gr.-B	Partial Differential Equation	29	20		
	PM4/13	Gr.-A	Computational Mathematics (Theory)	30	25	4	5 hr
		Gr.-B	(OP1)* Mathematical Logic	31	25		
			(OP2)* Number Theory	32	25		
			(OP3)* Distribution Theory	33	25		
			(OP4)* Calculus of Variation & Integral Equation	34	25		
			(OP5)* Automata Theory	35	25		
			(OP6)* Mechanics	36	25		
			(OP7)* Algebraic Geometry	37	25		
			(OP8)* Galois Theory	38	25		
	PM4/E1/101-111		Elective -I	2	50	4	5 hr
	PM4/E2/201-211		Elective -II	3	50	4	5 hr
	PM4/14/Pr		Computational Mathematics (Practical)	39	25	2	3 hr
	PM4/15		Dissertation, Internal Assessment, Seminar & Grand Viva		25	2	2 hr
			Total		**250**	**20**	**25 hr**
			Grand Total		**1000**	**80**	

***N.B. : For the Course PM4/13 Gr.-B, a student has to opt (subject to availability) for *any one* of the subjects from (OP1), (OP2), (OP3), (OP4), (OP5), (OP6), (OP7) and (OP8).**

Details of Code for Elective - I Courses**

Course Structure | Elective I | Elective II

****N.B. : A student has to opt (subject to availability) for *any one* of the subjects from above list.**

Course Structure | Elective I | Elective II

Details of Code for Elective - II Courses***

Course Structure | Elective I | Elective II

Serial No.	Course ID	Subject Code	Name of the Courses	Page Number	Full Marks
1	PM3/E2	201	Modules and Rings -I	64	50
	PM4/E2		Modules and Rings -II	65	50
2	PM3/E2	202	Advanced Functional Analysis -I	66	50
	PM4/E2		Advanced Functional Analysis -II	67	50
3	PM3/E2	203	Fourier Analysis -I	68	50
	PM4/E2		Fourier Analysis -II	69	50
4	PM3/E2	204	Rings of Continuous functions -I	70	50
	PM4/E2		Rings of Continuous functions -II	71	50
5	PM3/E2	205	Structures on Manifolds -I	72	50
	PM4/E2		Structures on Manifolds -II	73	50
6	PM3/E2	206	Advanced Number Theory -I	74	50
	PM4/E2		Advanced Number Theory -II	75	50
7	PM3/E2	207	Advanced General Topology -I	76	50
	PM4/E2		Advanced General Topology -II	77	50
8	PM3/E2	208	Theory of Linear Operators -I	78	50
	PM4/E2		Theory of Linear Operators -II	79	50
9	PM3/E2	209	Banach Algebra -I	80	50
	PM4/E2		Banach Algebra -II	81	50
10	PM3/E2	210	Non-standard Analysis -I	82	50
	PM4/E2		Non-standard Analysis -II	83	50
11	PM3/E2	211	Theory of Frames -I	84	50
	PM4/E2		Theory of Frames -II	85	50

*****N.B. : A student has to opt (subject to availability) for *any one* of the subjects from above list.**

Course Structure | Elective I | Elective II

Group Theory

Semester : I	Group : A
Course ID : PM1/01	Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- Isomorphism theorems of groups, external direct product and internal direct product of groups, direct product of cyclic groups, semi direct products, classification of all groups of order ≤ 12, group actions, Cayley's theorem, extended Cayley's theorem, Burnside theorem, conjugacy classes, class equation.
- Cauchy's theorem on finite groups, p-group, Centre of p-groups. Sylow's theorems, some applications of Sylow's theorems, Simple groups, nonsimplicity of groups of order p^n $(n > 1)$, pq, p^2q, p^2q^2 (p, q are primes), determination of all simple groups of order ≤ 60, nonsimplicity of A_n $(n \geq 5)$.
- Finite groups, structure theorem for finite Abelian groups, normal and subnormal series, composition series, Jordan–Hölder theorem, solvable groups and nilpotent groups.

References

[1] Malik, Mordeson and Sen; Fundamentals of Abstract Algebra; McGraw-Hill, 1997.

[2] T. W. Hungerford; Algebra; Springer, 1980.

[3] I. N. Herstein; Topics in Algebra; Wiley Eastern Ltd. New Delhi, 1975.

[4] Joseph J. Rotman; An introduction to the theory of groups; Springer-Verlag, 1990.

[5] S. Lang; Algebra (2nd ed.); Addition-Wesley.

[6] D. S. Dummit, R. M. Foote; Abstract Algebra, 2nd edition; Wiley Student edition.

[7] Michael Artin; Algebra; PHI. (Eastern Economy Edition) Prentice Hall.

[8] Saban Alaca, Kenneth S. Williams; Introduction to Algebraic Number Theory; Cambridge University Press.

Course Structure | Elective I | Elective II

Ring Theory

Semester : I Course ID : PM1/01	Group : B Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- Ideal, quotient ring, ring embeddings, Euclidean domain, principal ideal domain, prime elements and irreducible elements, maximal ideals, maximal ideals in some familiar rings of functions, maximal ideals space of a ring, prime ideals, primary ideals.
- Polynomial ring and factorization of polynomials over a commutative ring with identity, the division algorithm in $K[x]$ where K is a field, $K[x]$ as Euclidean domain, unique factorization domain (UFD), if D is UFD then so are $D[x]$ and $D[x_1, x_2, \ldots, x_n]$.
- Eisenstein's criterion of irreducibility, Noetherian and Artinian rings, Hilbert Basis Theorem.
- Ring of fractions.
- **Application of techniques of groups and rings to prove some theorems in number theory :** Fermat's Theorem, Euler's Theorem, Willson's Theorem, Chinese Remainder Theorem.

References

[1] Malik, Mordeson and Sen; Fundamentals of Abstract Algebra; McGraw-Hill, 1997.

[2] T. W. Hungerford; Algebra; Springer, 1980.

[3] I. N. Herstein; Topics in Algebra; Wiley Eastern Ltd. New Delhi, 1975.

[4] Joseph J. Rotman; An introduction to the theory of groups; Springer-Verlag, 1990.

[5] S. Lang; Algebra (2nd ed.); Addition-Wesley.

[6] D. S. Dummit, R. M. Foote; Abstract Algebra, 2nd edition; Wiley Student edition.

[7] Michael Artin; Algebra; PHI. (Eastern Economy Edition) Prentice Hall.

[8] Saban Alaca, Kenneth S. Williams; Introduction to Algebraic Number Theory; Cambridge University Press.

[9] Michael Francis Atiyah and I. G. MacDonald; Introduction to Commutative Algebra; Addison-Wesley Series in Mathematics, 1969.

Course Structure | Elective I | Elective II

Real Analysis – I

Semester : I Course ID : PM1/02	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Metric Spaces :** Baire Category theorem, completion of metric spaces, Banach contraction principle and some of its applications, compactness, total boundedness, characterization of compactness for arbitrary metric spaces, equicontinuity, Arzella-Ascoli theorem, Weierstrass approximation theorem, Stone-Weierstrass theorem.
- Lebesgue outer measure, Lebesgue measurable sets, Borel sets, approximation of Lebesgue measurable sets by topologically nice sets, non-measurable sets, Cantor sets.
- Lebesgue measurable functions, algebra of measurable functions, limit of sequence of measurable functions, simple functions, measurable functions as point-wise limit of sequence of simple functions, approximation of Lebesgue measurable functions by continuous functions, Luzin's theorem.
- Lebesgue integral, monotone convergence theorem, Fatou's lemma, dominated convergence theorem, properties of Lebesgue integrable functions, relation between Lebesgue integral and Riemann integral.
- Absolutely continuous functions, properties of absolutely continuous functions, characterisation of absolutely continuous functions in the context of Lebesgue integration [Fundamental theorem of Lebesgue integral]

Further Reading :

- Various algebraic structures of measurable/integrable functions.
- Cantor-like sets.
- Non-Borel Lebesgue measurable sets and functions.
- Semi-continuous functions.
- Double sequences, double series, Stolz's theorem, double series of positive terms, absolute convergence of double series.
- Vitali-covering theorem.

References

[1] T. M. Apostol : Mathematical Analysis; Addison-Wesley Publishing Co. 1957.

[2] A. Bruckner, J. Bruckner & B. Thomson : Real Analysis; Prentice Hall, 1997.

[3] T. J. I' A. Bromwitch : Infinite Series; MacMillan, London, 1949.

[4] C. Goffman : Real Functions; Holt, Rinehart and Winston, N.Y, 1953.

[5] J. F. Randolph : Basic Real and Abstract Analysis; Academic Press, N.Y, 1968.

[6] P. K. Jain and K. Ahmad : Metric Spaces, Narosa Publishing House.

[7] W. Rudin : Principles of Mathematical Analysis; McGraw-Hill, N.Y, 1964.

[8] E. Hewitt and K. Stromberg : Real and Abstract Analysis; John Wiley, N.Y., 1965.

[9] G. De. Barra; Measure Theory & Integration; Wiley Eastern Limited, 1987.

[10] Charles Schwartz; Measure, Integration & Function Spaces; World Scientific, 1994.

[11] Inder Kumar Rana; An Introduction to measure & Integration; Narosa Publishing House, 1997.

[12] P. R. Halmos; Measure Theory; D.Van Nostrand Co. inc. London, 1962.

[13] P. K. Jain & V. P. Gupta; Lebesgue Measure & Integration; New Age International(P)limited Publishing Co, New Delhi, 1986.

[14] H. L. Royden; Real Analysis; Macmillan Pub.Co.inc 4th Edition, New York, 1993.

[15] Walter Rudin; Real and Complex Analysis; Tata McGraw Hill Publishing Co limited, New Delhi, 1966.

Course Structure | Elective I | Elective II

Complex Analysis - I

Semester : I	Group : A
Course ID : PM1/03	Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- **Analytic functions :** Definition of analytic function, functions represented by power series as natural examples of analytic functions, the functions $\sin z, \cos z, \exp z$ and $\log z$, branches of logarithmic functions, Möbius transformations as special example of analytic functions and some properties of Möbius transformation.

- **Complex Integration :** Line integral of complex functions and its basic properties, winding number of a closed rectifiable curve about points in $\mathbb{C}$, Cauchy-Goursat theorem, Cauchy's integral formula, Cauchy's integral formula for derivatives, Morera's theorem, Liouville's theorem, fundamental theorem of algebra, Maximum Modulus theorem and its applications, Mean Value property of an analytic function, Mean Value property and Maximum Modulus principle of a harmonic function on a region, power series representation of analytic function, zeros of an analytic function, Schwarz lemma, Schwarz-Pick Lemma, interior uniqueness theorem/Identity theorem, open mapping theorem.

References

[1] R. P. Agarwal, K. Perera and S. Pinelas; An Introduction To Complex Analysis; Springer-Verlag, 2011.

[2] L. V. Ahlfors; Complex Analysis; McGraw-Hill; New York, 1979 (Third Edition).

[3] R. V. Churchill and J. W. Brown; Complex Variables and Applications; McGraw-Hill; New York, 1996.

[4] J. B. Conway; Functions of One Complex Variable; Narosa Publishing, New Delhi, 1973.

[5] S. Lang; Complex Analysis, Fourth edition; Springer-Verlag, 1999.

[6] A. I. Markushivich; Theory of Functions of Complex Variables, Vol-I, II; Prentice-Hall, 1965.

[7] R. Narasimhan; Complex Analysis in one variable; Birkhauser, Boston, 1984.

[8] S. Ponnusamy; Foundations of Complex Analysis; Narosa Publishing; New Delhi, 1973.

[9] H. A. Priestly; Introduction to complex analysis; Clarendon Press, Oxford, 1990.

[10] E. M. Stein and R. Shakarchi; Complex Analysis; Princeton University Press, Princeton, New Jersey, 2003

Course Structure | Elective I | Elective II

Ordinary Differential Equation

Semester : I	Group : B
Course ID : PM1/03	Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- **Existence and Uniqueness of Initial value problems :** Lipschitz condition, successive approximations and Picard's theorem, dependence of solutions on the initial conditions, dependence of solutions on the functions, continuation of the solutions and maximal interval of existence.
- **Linear Differential Equations :** Basic theory of the n-th order homogeneous and non-homogeneous linear differential equation, Wronskian and its properties, fundamental solutions, Sturm- Liouville problem, finding of eigen values and eigen function of Sturm-Liouville problem, orthogonality of eigen functions, the expansion of a function in a series of orthonormal eigen functions, Green's function.
- **Non-linear Differential Equations :** Phase plane, paths and critical points, critical points and stability of linear systems, paths of linear systems, limit cycles and periodic solutions, stability of the critical points of non-linear systems and their equivalence with the corresponding linearized system.

Further Reading :

- Linear Differential equations on complex domain.
- Ordinary points and regular singular points, series solution.
- Hypergeometric, Legendre and Bessel equations, Legendre polynomials, Hermite polynomials, Bessel functions of first kind.

References

[1] S. L. Ross; Introduction to ordinary differential equations; John-Wiley, New York, 1989.

[2] G. F. Simmons; Differential equations with applications and historical notes; Tata McGraw Hill, New Delhi, 1976.

[3] W. E. Boyce & R. C. Diprima; Elementary differential equations and boundary value problems; John Wiley & Sons, New York, 1977.

[4] E. A. Coddington; An Introduction to Ordinary Differential Equations; PHI Learning 1999.

[5] P. Hartman; Ordinary Differential Equations; John Wiley and sons, New York, 1964.

[6] M. Hirsch, S. Smale and R. Deveney; Differential Equations, Dynamical Systems and Introduction to Chaos; Academic Press, 2004.

[7] L. Perko; Differential Equations and Dynamical Systems; Texts in Applied Mathematics, Vol. 7, 2nd ed., Springer Verlag, New York, 1998.

[8] D. A. Sanchez; Ordinary Differential Equations and Stability Theory : An Introduction; Dover Publ. Inc., New York, 1968.

Course Structure | Elective I | Elective II

General Topology - I

Semester : I	Group : A
Course ID : PM1/04	Full Marks : 30
Minimum number of classes required : 40	

Course Structure | Elective I | Elective II

- Definition and examples of topological spaces, closed sets, closure, dense subsets, neighbourhood, interior, exterior and boundary, accumulation point, derived set, bases and subbases, subspace topology, finite product of topological spaces, alternative methods for defining a topology in terms of Kuratowski closure operator and neighbourhood system.
- Open, closed and continuous functions and homeomorphism, topological invariants, isometry and metric invariants.
- Countability Axioms : First and second countability, separability and Lindelöf property.
- Separation Axioms : T_i-property ($i = 0, 1, 2, 3, 3\frac{1}{2}, 4, 5$), regularity, complete regularity, normality and complete normality; their characterizations and basic properties, Urysohn's lemma, Tietze's extension theorem, T_5-property of a metric space.

References

[1] N. Bourbaki; General Topology Part-I (Transl.); Addison Wesley, Reading(1966).

[2] J. Dugundji; Topology; Allyn and Bacon, Boston,1966(Reprinted in India by Prentice Hall of India Pvt. Ltd.).

[3] R. Engelking; General Topology; Polish Scientific Publishers, Warsaw (1977).

[4] J. G. Hocking and C. S. Young; Topology; Addison-Wesley, Reading (1961).

[5] S. T. Hu; Elements of General Topology; Holden-Day, San Francisco (1964).

[6] K. D. Joshi; Introduction to Topology; Wiley Eastern Ltd. (1983).

[7] J. L. Kelley; General Topology; Van Nostrand, Princeton (1955).

[8] M. J. Mansfield; Introduction to Topology; D-van Nostrand Co. Inc, Princeton N.Y. (1963).

[9] B. Mendelson; Introduction to Topology; Allyn and Becon Inc, Boston (1962).

[10] James R. Munkress; Topology (2nd edit.); Pearson Education (2004).

[11] W. J. Pervin; Foundations of General Topology; Academic Press, N.Y. (1964).

[12] George F. Simmons; Introduction to Topology and Modern Analysis; McGraw-Hill, N.Y.(1963).

[13] L. Steen and J. Seebach; Counterexamples in Topology; Holt, Rinechart and Winston, N.Y. (1970).

[14] W. J. Thron; Topological Structures; Holt, Rinehart and Winston, N.Y. (1966).

[15] Stephen Willard; General Topology; Addison-Wesley, Reading (1970).

Course Structure | Elective I | Elective II

Differential Geometry of Curves and Surfaces

Semester : I	Group : B
Course ID : PM1/04	Full Marks : 20
Minimum number of classes required : 30	

Course Structure | Elective I | Elective II

- Curves in plane and space, arc-length, reparametrization, closed curves, level curves versus parametrized curves, curvature.
- Some global properties of curves : simple closed curve, isoperimetric inequality, Four Vertex theorem.
- Regular surfaces, differential functions on surfaces, the tangent plane and the differential maps between regular surfaces, the first fundamental form, normal fields and orientability.
- Gauss map, shape operator, the second fundamental form, normal and principle curvatures, Gaussian and mean curvatures.

Further Reading :

- **Tensors :** Different transformation laws, properties of tensors, metric tensor, Riemannian space, covariant differentiation, Einstein space, curves in space : intrinsic differentiation, parallel vector fields, Serret- Frenet formulii.
- **Surface :** First fundamental form, angle between two intersecting curves on a surface, Geodesic, Geodesic curvature, Gaussian curvature, developable surface.
- **Surface in Space :** Tangent and normal vector on a surface, second fundamental form, Gauss's formula, Weingarten formula, third fundamental form, Gauss and Codazzi equations, principal curvature, lines of curvature, asymptotic lines.

References

[1] Andrew Pressley; Elementary Differential Geometry; Springer, 2010.

[2] Barrett O'Neill; Elementary Differential Geometry; Elsevier, 2006.

[3] Christian Br; Elementary Differential Geometry; Cambridge University Press, 2011.

[4] Manfredo P. Do Carmo; Differential Geometry of Curves and Surfaces; Prentice-Hall, Inc., Upper Saddle River, New Jersey 07458, 1976.

[5] L. P. Eisenhart; An Introduction to Differential Geometry (with the use of tensor Calculus); Princeton University Press, 1940.

[6] I. S. Sokolnikoff; Tensor Analysis, Theory and Applications to Geometry and Mechanics of Continua, 2nd Edition; John Wiley and Sons., 1964.

[7] B. Spain; Tensor Calculus; John Wiley and Sons, 1960.

[8] M. Spivak; A Comprehensive Introduction to Differential Geometry, Vols I-V; Publish or Perish, Inc. Boston, 1979.

Course Structure | Elective I | Elective II

Discrete Mathematics - I

Semester : I	Group : A
Course ID : PM1/05	Full Marks : 30
Minimum number of classes required : 40	

Course Structure | Elective I | Elective II

Graph Theory :

- Definition of an undirected graph, degree of a vertex, historical background of Graph Theory.
- Walks, paths, trails and cycles, subgraphs and induced subgraphs, connectivity, distance in a graph, complete and complete bipartite graphs.
- Eulerian graphs, Euler theorem on existence of Euler paths and circuits, Hamiltonian paths and cycles, Hamiltonian graphs.
- Definition and properties of trees, minimal spanning tree in a weighted graph, Kruskal algorithm and Primp's algorithm.
- Definition of planar graphs, Kuratowski's two graphs, the Euler polyhedron formula, Euler identity for connected planar graphs, detection of planarity, Kuratowski's theorem (proof not required).
- Directed graphs (digraphs), digraphs and binary relations, strongly connected digraphs, Euler digraphs, vertex colouring of graphs, Chromatic number of graphs and its elementary properties, matrix representation of graphs, adjacency matrices of graphs and digraphs and their properties, path matrix, incidence matrices of graphs and digraphs and their properties.

References

[1] N. Deo; Graph Theory with Application to Engineering and Computer Science; Prentice Hall of India, New Delhi, 1990.

[2] John Clark and Derek Allan Holton; A First Look at Graph Theory; World Scientific, New Jersey, 1991.

[3] F. Harary; Graph Theory; Narosa Publishing House, New Delhi, 2001.

[4] J. A. Bondy and U. S. R. Murty; Graph theory and related topics; Academic Press, New York, 1979.

Course Structure | Elective I | Elective II

Multivariate Calculus

Semester : I	Group : B
Course ID : PM1/05	Full Marks : 20
Minimum number of classes required : 30	

Course Structure | Elective I | Elective II

- **Multivariable Differential Calculus :** Introduction, directional derivative and continuity, total derivative, total derivative expressed in terms of partial derivatives, Jacobian matrix, chain rule, matrix form of the chain rule, Mean Value theorem for differentiable functions, a sufficient condition for differentiability, a sufficient condition for equality of mixed partial derivatives, Taylor's formula for functions from $\mathbb{R}^n$ to $\mathbb{R}^1$.
- **Implicit Functions and Extremum Problems :** Introduction, inverse function theorem, implicit function theorem, extrema of real-valued functions of several variables.

Further Reading :

- **Multiple Integrals :** Partitions of rectangles and step functions, double integral, double integral as volume, integrability of functions, applications to area and volume, Pappus's theorem, Green's theorem and its applications, change of variables and transformation formula.
- **Surface Integrals :** Surface, fundamental vector product, area of a parametric surface, surface integrals, Stoke's theorem, curl and divergence of a vector field, divergence theorem.

References

[1] T. M. Apostol; Mathematical Analysis; Narosa Publishing House, New Delhi.

[2] M. Spivak; Calculus on Manifolds; W. A Benjamin, New York, 1965.

[3] C. Goffman; Calculus of Several Variables; A Harper International Student reprint, 1965.

[4] W. Rudin; Principles of Mathematical Analysis; McGraw-Hill, New York, 1964.

Course Structure | Elective I | Elective II

Linear Algebra

Semester : II Course ID : PM2/06	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- $PA = LU$ and LDU factorization of a matrix, its application in solving $Ax = b$, rank factorization of a matrix, rank cancellation.
- Fundamental theorem of Linear Algebra, part I and II, existence and uniqueness of solutions to $Ax = b$, a matrix transforms its row space to its column space.
- Matrix of orthogonal projection, least square solution of overdetermined system $Ax = b$, Moore-Penrose inverse [through rank factorization].
- Duality and transposition, linear forms or linear functionals, dual space V^d, bi-dual space V^{dd}, dual basis, natural isomorphism between V^d and V^{dd}, Annihilators W° of a nonempty subset W of a vector space V, $\dim W^\circ = \dim V - \dim W$, transpose T^t of a linear transformation T, $T^{tt} = T$, $(T^t)^{-1} = (T^{-1})^t$ if T is an isomorphism, $(\text{Im } T)^\circ = \text{Im } T^t$, $\dim \text{Im } T = \dim \text{Im } T^t$, $\dim \ker T = \dim \ker T^t$.
- Eigenvalues and eigenvectors, characteristic polynomial of a linear transformation, eigenvalues and eigenvectors of a linear transformation, diagonalisation, annihilating polynomials, invariant subspace, simultaneous trangulisation and diagonalization, direct sum decomposition, invariant direct sum, primary decomposition theorem.
- Linear transformations on a finite dimensional inner product spaces, Riesz representation of the linear functional on inner product space, adjoint T^* of a linear operator $T : V \longrightarrow V$, matrix representation of T^*, normal and self-adjoint operators, eigen values of a self-adjoint operator are real, unitary and orthogonal operators and their matrices, orthogonal projections, the spectral theorem and its consequences.
- Modules over a ring with identity, submodules, operations on submodules, quotient modules and module homomorphisms.
- Cyclic modules, finitely generated modules, free modules.
- Modules over PID, Fundamental Structure Theorem for finitely generated modules over a PID and its applications to finitely generated Abelian groups, rational canonical form and Jordan canonical form of a linear transformation.

Further Reading :

- Vector space of infinite dimension, existence of bases, extension of a linearly independent subset to a basis, reduction of a generating subset to a basis, characterizations of basis as a maximal linearly independent subset and as a minimal generating subset, equality of cardinalities of any two bases.
- Bilinear and Quadratic forms : Matrix representation of a bilinear form, symmetric bilinear forms, a bilinear form is symmetric if and only if its matrix representation is symmetric, diagonalization of the bilinear and symmetric bilinear forms, orthogonal diagonalization of a real quadratic form, positive definite negative definite and semidefinite real quadratic forms, rank, signature and index of a quadratic form, reduction of a quadratic form to normal form, Sylvester's law of inertia, simultaneous reduction of two quadratic forms.
- Geometric significance of a determinant.
- Calculation of Fourier Coefficients in the light of linear algebra [actually in the light of orthogonal projection].

References

[1] Friedberg, Insel and Spence; Linear Algebra; Prentice Hall of India.

[2] S. Kumaresan; Linear Algebras, a geometric approach; Prentice Hall of India, 2001.

[3] Hoffman and Kunze; Linear Algebra; Prentice Hall of India, New Delhi.

[4] Liptschutz; Linear Algebra; McGraw Hill.

[5] M. Artin; Algebra; Prentice Hall of India, 1991.

[6] Gilbert Strang; Introduction to Linear Algebra; Fifth Edition, Wellesley-Cambridge Press and SIAM, 2016.

[7] J. H. Kwak and S. Hong; Linear Algebra (2nd Edition); Birkhuser, 2004.

[8] Y. Ju, W. Xing, C. Lin, J. Hu, F. Wang; Linear Algebra : Theory and Applications; CENGAGE Learning and Tsinghua University Press, 2010.

[9] D. S. Dummit, R. M. Foote; Abstract Algebra, 2nd Edition; Wiley Student Edition.

[10] Vivek Sahai and Vikas Bist; Linear Algebra, 2nd Edition; Narosa, New Delhi, 2013.

Course Structure | Elective I | Elective II

Real Analysis - II

Semester : II	Group : A
Course ID : PM2/07	Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- **Cardinal Number :** Concept of Cardinal number of an infinite set, order relation of Cardinal numbers, Schröder-Bernstein theorem, the set 2^A, Axiom of choice, arithmetic of Cardinal numbers, Cardinality of Cantor set, continuum hypothesis.
- Everywhere continuous but nowhere differentiable functions.
- Abstract measure space and integration on abstract measure space [analogous generalisation from real context]
- Integration on product measure, Fubini's theorem.
- Minkowski's and Hölder's inequality, L^p $(1 \leq p \leq \infty)$ space and its completeness.

Further Reading :

- Study of everywhere continuous nowhere differentiable functions in the context of Category theory.
- Construction of Lebesgue measure using the well known Caratheodory extension theorem.
- **Various types of convergence and their comparison :** Almost uniform convergence, convergence in measure, convergence in mean, almost everywhere convergence, Egoroff's theorem, Riesz's theorem interrelating convergence in measure and point-wise convergence.
- Complex measure, absolute continuity of measure, Radon-Nikodym theorem and its consequences.
- Fourier Series : Trigonometric polynomials, Fourier coefficients, convolution, Riemann-Lebesgue lemma, Plancherel identity, Dirichlet kernel, Fejer kernel, summability and convergence of Fourier series, interpolation theorems, Hausdörff-Young inequality.

References

[1] A. M. Bruckner, J. Bruckner & B. Thomson : Real Analysis, Prentice-Hall, N.Y. 1997.

[2] R. R. Goldberg : Methods of Real Analysis, Oxford-IBH, New Delhi, 1970.

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[4] E. Hewitt and K. Stromberg : Real and Abstract Analysis, John-Willey, N.Y. 1965.

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[13] Walter Rudin; Real and Complex Analysis; Tata Mcgraw Hill Publishing Co limited, New Delhi, 1966.

[14] G. B. Folland; Real Analysis : Modern Techniques and Their Applications;

[15] Stein, Shakarchi; Real Analysis : Measure Theory, Integration and Hilbert Spaces;

Course Structure | Elective I | Elective II

Complex Analysis-II

Semester : II	Group : B
Course ID : PM2/07	Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- Various kinds of singularities of complex valued functions, removable singularity, pole, essential singularity, classification of singularities using Laurent series development, Casorati-Weierstrass theorem concerning the nature of a function having an essential singularity, meromorphic functions, Residue theorem, contour integration and some applications, Argument Principle, Rouche's theorem, fundamental theorem of algebra as a corollary to Rouche's theorem.
- Analytic continuation and some basic properties, Analytic continuation via Reflection.
- Space of continuous functions, space of analytic functions and space of meromorphic functions defined over open connected domain in $\mathbb{C}$ — a few interesting properties : Arzela-Ascoli theorem, Hurwit'z theorem, Montel's theorem, a subspace of analytic functions is compact iff it is closed and locally bounded, Riemann mapping theorem.

References

[1] R. P. Agarwal, K. Perera and S. Pinelas; An Introduction To Complex Analysis; Springer-Verlag, 2011.

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[3] R. V. Churchill and J. W. Brown; Complex Variables and Applications: McGraw-Hill; New York, 1996.

[4] J. B. Conway; Functions of One Complex Variable; Narosa Publishing, New Delhi, 1973.

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[6] A. I. Markushivich; Theory of Functions of Complex Variables, Vol-I, II; Prentice-Hall, 1965.

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[8] S. Ponnusamy; Foundations of Complex Analysis; Narosa Publishing; New Delhi, 1973.

[9] H. A. Priestly; Introduction to complex analysis; Clarendon Press, Oxford, 1990.

[10] E. M. Stein and R. Shakarchi; Complex Analysis; Princeton University Press, Princeton, New Jersey, 2003.

Course Structure | Elective I | Elective II

General Topology - II

Semester : II Course ID : PM2/08	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Compactness :** Characterizations and basic properties, Alexander subbase theorem, compactness and separation axioms, compactness and continuous functions, sequentially, Frechet and countably compact spaces, compactness in metric spaces.
- **Connectedness :** Connected sets and their characterizations, connectedness of the real line, components, totally disconnected space, locally connected space, path connectedness, path components, locally path connected space.
- **Nets and Filters :** Convergence and cluster points, Hausdörffness, continuity, limit point of sets and compactness in terms of them, canonical way of converting nets to filters and vice-versa, ultrafilter, subnets and ultranet.
- **Product Topology :** Tychönoff product topology in terms of standard sub-base and its characterizations, projection maps, product spaces vis-à-vis separation axioms, 1st and 2nd countability, separability, Lindelöfness, connectedness, local connectedness, path connectedness and compactness (Tychönoff theorem), embedding lemma and Tychönoff embedding theorem.
- **Identification Topology and Quotient Spaces :** Definitions and examples of quotient topology and quotient maps, definition of quotient space of a space X determined by an equivalence relation on X and associated theorems, cones and suspensions as examples, divisible properties.
- **Compactification :** Local compactness and one-point compactification, Stone-Čech compactification.

Further Reading :

- **Metrizations :** The Urysohn metrization theorem, the Nagata-Smirnov metrization theorem.
- **Uniform Space :** Uniformity, topology through uniformity, metric space as uniform space, characterisation of uniform space by means of separation property, concept of uniform continuity, Cauchy sequence and Cauchy net, completeness of uniform space.
- **Paracompact Space :** Some basics of Paracompact Space.

References

[1] N. Bourbaki; General Topology Part-I (Transl.); Addison Wesley, Reading(1966).

[2] J. Dugundji; Topology; Allyn and Bacon, Boston,1966(Reprinted in India by Prentice Hall of India Pvt. Ltd.).

[3] R. Engelking; General Topology; Polish Scientific Publishers, Warsaw (1977).

[4] J. G. Hocking and C. S. Young; Topology; Addison-Wesley, Reading (1961).

[5] S. T. Hu; Elements of General Topology; Holden-Day, San Francisco (1964).

[6] K. D. Joshi; Introduction to Topology; Wiley Eastern Ltd. (1983).

[7] J. L. Kelley; General Topology; Van Nostrand, Princeton (1955).

[8] M. J. Mansfield; Introduction to Topology; D-van Nostrand Co. Inc, Princeton N.Y. (1963).

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[11] W. J. Pervin; Foundations of General Topology; Academic Press, N.Y. (1964).

[12] George F. Simmons; Introduction to Topology and Modern Analysis; McGraw-Hill, N.Y.(1963).

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[14] W. J. Thron; Topological Structures; Holt, Rinehart and Winston, N.Y. (1966).

[15] Stephen Willard; General Topology; Addison-Wesley, Reading (1970).

Course Structure | Elective I | Elective II

Functional Analysis

Semester : II Course ID : PM2/09	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Normed linear space (n.l.sp.), Banach space with examples, quotient space.
- Bounded linear transformation, its equivalence with continuity, space of bounded linear transformations, equivalence of two norms in a linear space, equivalence of any two norms in a finite dimensional vector space, other important properties of a finite dimensional n.l.sp.
- Bounded linear functionals on various n.l.sp., Hahn-Banach theorems and consequences, dual and 2nd dual of a n.l.sp., separability and reflexivity of n.l.sp.
- Open mapping theorem, closed graph theorem and uniform boundedness principle, some applications of these theorems.
- Weak and weak*-convergence, Banach-Alaoglu theorem.
- Inner product space, Hilbert space, orthonormality, orthogonal complement, orthonormal basis, Bessel's inequality, Parseval's equation, Gram-Schmidt orthonomalisation process, Riesz representation theorem, reflexivity of Hilbert space, separable and non-separable Hilbert space.
- Introduction to operator theory :
  - ◇ Compact operator and its characterisation, space of compact operators, weak-convergence and compact operator, rank of compact operator.
  - ◇ Adjoint of an operator on Hilbert space, properties of adjoint operation, self-adjoint operator and its characterisation, positive operator and non-singularity, concept of normal operator and its characterisation, unitary operator and its characterisation.

Further Reading :

- Determination of dual of some familiar normed linear spaces.
- General model of all Hilbert spaces (up to isometric isomorphism) — $\ell^2(S)$, for any nonempty set S.
- Introduction to spectral theory : Resolvent set, spectrum and spectral radius of operators on Banach space, spectral mapping theorem for polynomials.

References

[1] Bachman and Narici ; Functional Analysis ; Academic Press (1966).

[2] G. F. Simmons ; Introduction to Topology and Modern Analysis ; McGraw-Hill Book Company (1963).

[3] Goffman and Pedrick ; First Course in Functional Analysis ; Prentice-Hall, Inc.

[4] Walter Rudin ; Functional Analysis ; Tata McGraw-Hill (1974).

[5] John B. Conway; A Course in Functional Analysis; Springer, (1990).

[6] A. E. Taylor ; Introduction to Functional Analysis ; John Wiley & Sons. (1958).

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[9] Jain, Ahuja and Ahmad ; Functional Analysis ; New Age International (P) Ltd. (1997).

Course Structure | Elective I | Elective II

Discrete Mathematics-II

Semester : II	Group : A
Course ID : PM2/10	Full Marks : 20
Minimum number of classes required : 30	

Course Structure | Elective I | Elective II

- **Ordered Sets :** Partially ordered sets (posets), Hasse diagram of partially ordered sets, linear orders, linear extension of a partially ordered set, Realizer and dimension of a poset, lattices and their properties, complete lattice, sublattices, lattice as a partially ordered set, bounded lattice, distributive lattice, complements and completed lattices.
- The pigeonhole principle and its simple applications.
- **Recurrence relations and Generating Functions :** Introduction, recurrence relations, methods of solving recurrence relation with constant coefficients, solution of recurrence relations using Generating Functions.

References

[1] N. Deo; Graph Theory with Application to Engineering and Computer Science; Prentice Hall of India, New Delhi, 1990.

[2] John Clark and Derek Allan Holton; A First Look at Graph Theory; World Scientific, New Jersey, 1991.

[3] D. S. Malik and M. K. Sen; Discrete mathematical structures : theory and applications; Thomson, Australia, 2004.

[4] Edward R. Scheinerman; Mathematics A Discrete Introduction; Thomson Asia Ltd., Singapore, 2001.

Course Structure | Elective I | Elective II

Theory of Manifolds

Semester : II	Group : B
Course ID : PM2/10	Full Marks : 30
Minimum number of classes required : 40	

Course Structure | Elective I | Elective II

- Topological manifolds – examples, differentiable manifolds – examples, smooth maps and diffeomorphisms, derivatives of smooth maps, local expression for the differential, curves in a manifold, immersion and submersion, rank, critical and regular points, submanifolds and regular submanifolds.
- Tangent space and cotangent space, vector fields on a manifold, Lie algebra of vector fields on a manifold, integral curves of a vector field, local flows, f-related vector fields, Lie bracket, 1- parameter group of transformations, tangent bundles, manifold structure on tangent bundle, vector bundles.
- Differential forms, local expression for a k-form, pull back of a k-form, wedge product, exterior differentiation, existence and uniqueness of exterior differentiation on manifold, exterior differentiation under pull-back.

Further Reading :

- Quotient manifold, examples of quotient manifolds.
- Lie group, examples of Lie groups, action of a Lie group on a manifold, transformation group, action of a discrete group on manifold, invariant forms on a Lie group.
- Lie derivative of vector fields, Lie derivatives of differential forms, Frobenius theorem.
- Orientations on manifold, orientations and differential forms, manifolds with boundary.
- Integration on manifolds – Stoke's theorem, line integral and Green's theorem.
- **Applications to physical systems :** Thermodynamics, Hamiltonian mechanics, electromagnetism, dynamics of a perfect fluid and cosmology.

References

[1] W. M. Boothby; An Introduction to Differentiable Manifolds and Riemanian Geometry; Academic Press, Revised, 2003.

[2] L. Conlon; Differentiable Manifolds, A First Course; Birkhauser (Second Edition), 2008.

[3] W. D. Curtis and F. R. Miller; Differential Manifolds and Theoretical Physics; Academic Press, 1985.

[4] S. Helgason; Differential Geometry, Lie Groups and Symmetric Spaces; Academic Press, 1978.

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[7] S. Kumaresan; A course in Differential Geometry and Lie-groups; Hindustan Book Agency.

[8] S. Lang; Differential and Riemannian manifolds; Springer-Verlag, 1995.

[9] John M. Lee; Introduction to smooth manifolds; Springer.

[10] S. Morita; Geometry of Differential forms; American Mathematical Society.

[11] Bernard Schutz; Geometrical Methods of Mathematical Physics; Cambridge University Press, 1980.

[12] M. Spivak; A Comprehensive Introduction to Differential Geometry, volumes 1 and 2; Publish or Perish, 1979.

[13] L. Tu; An Introduction to Manifolds; 2nd edition, 2011.

[14] K. Yano and M. Kon; Structure on Manifolds; World Scientific, 1984.

[15] Robert H. Wasserman; Tensor and manifolds with applications (2nd Edition); Oxford University Press, 2009.

Course Structure | Elective I | Elective II

Field Extension

Semester : III	Group : A
Course ID : PM3/11	Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- **Field Extensions :** Algebraic extensions, transcendental extensions, degree of extensions, simple extensions, finite extensions, simple algebraic extensions, minimal polynomial of an algebraic element, isomorphism extension theorem.
- **Splitting fields :** Fundamental theorem of general algebra (Krönekar theorem), existence theorem, isomorphism theorem, algebraically closed field, existence of algebraically closed field, algebraic closures, existence and uniqueness (up to isomorphism) of algebraic closures of a field, field of algebraic members.
- Normal extensions, separable and inseparable polynomials, separable and inseparable extensions, perfect field, Artin's theorem.

References

[1] Malik, Mordeson and Sen; Fundamentals of Abstract Algebra; McGraw Hill (1997).

[2] J. N. Herstein; Topics in Algebra; Wiley Eastern Ltd. 1975.

[3] I. Stewart; Galois Theory; Chapman and Hall 1989.

[4] J. P. Escofier; Galois Theory; GTM Vol 204. Springer 2001.

Course Structure | Elective I | Elective II

Algebraic Topology - I

Semester : III	Group : B
Course ID : PM3/11	Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

Homotopy Theory :

- Homotopy between continuous maps, homotopy relative to a subset, homotopy class, null homotopy, contractibility of spaces, homotopy equivalent spaces, homotopy properties.
- Deformability, deformation retracts, strong deformation retracts, homotopy between paths, product of paths, fundamental group $\Pi(X, x)$ of a space X based at the point $x \in X$, induced homomorphism and related properties, simply connected space, special Van Kampan theorem and fundamental group of $S^n\,(n \geq 2)$.
- Fundamental Group of S^1, fundamental group of the product and of Torus, $\mathbb{R}^2$ and $\mathbb{R}^n\,(n > 2)$ are not homeomorphic.
- Fundamental theorem of algebra and Brouwer fixed point theorem, covering projection, covering spaces, lifting of paths and homotopies, the fundamental group of a covering space. the Monodromy theorem, the Borsuk-Ulam theorem and Ham-Sandwich theorem.

References

[1] A. Dold; Lectures on Algebraic Topology; Springer-Verlag (1972).

[2] W. Fulton; Algebraic Topology : A First Course; Springer-Verlag (1995).

[3] M. Greenberg; Lectures on Algebraic Topology; W.D.Benjamin, N.Y. (1967).

[4] Allen Hatcher; Algebraic Topology; Cambridge Univ. Press (2002).

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[7] James R. Munkres; Topology (2nd Edit.); Pearson Education Inc. (2004).

[8] E. H. Spanier; Algebraic Topology; McGraw Hill Book Co. N.Y. (1966).

[9] C. T. C. Wall; A Geometric Introduction to Topology; Addison-Wesley Publ. Co. Inc(1972).

Course Structure | Elective I | Elective II

Algebraic Topology - II

Semester : IV	Group : A
Course ID : PM4/12	Full Marks : 30
Minimum number of classes required : 40	

Course Structure | Elective I | Elective II

Homology Theory :

- **Elements of simplicial homology :** Barycentric co-ordinates, simplex, geometric complexes and polyhedrons, simplicial mappings and simplicial approximation theorem. Oriented complexes, incidence numbers, chains, cycles and boundaries; Simplicial homology groups, computation of simplicial homology groups, the decomposition theorems for abelian groups, Betti numbers and torsion coefficients.
- **Singular homology :** computation of singular homology groups, Mayer-Vietoris sequence; Homotopy invariance; Equivalence of simplicial and singular homology; Relation between fundamental group and first homology group.
- **Applications :** Borsuk Ulam theorem, Brouwer's no-retraction theorem, Brouwer fixed point theorem, Invariance of dimensions, etc.

Further Reading :

- Relative homology and Excision theorem.
- CW-complex, sub-complex and CW-pairs; Euler characteristic, Euler- Poincare theorem. Simplicial approximation to CW-complex. Computation of homology of CW-complex.
- Degree of a map from a sphere to itself; Cellular homology of a CW-complex; Isomorphism between singular and cellular homology of a CW-complex.
- Homology with coefficient and Universal coefficient theorem.
- Kunneth formula for homology of the product of two spaces.

References

[1] G.E.Bredon; Topology and Geometry; Springer-Verlag GTM 139 (1993).

[2] A.Dold; Lectures on Algebraic Topology; Springer-Verlag (1980).

[3] W.Fulton; Algebraic Topology, A First Course; Springer-Verlag (1995).

[4] M. J. Greenberg and J. R. Harper; Algebraic Topology : A first course; Perseus Books (Mathematical Lecture Notes series) (1981).

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[6] William S.Massey; A Basic Course in Algebraic Topology; Springer-Verlag, New York Inc.(1993).

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[10] James W. Vick; Homology Theory : An introduction to Algebraic Topology; Springer-Verlag, N. Y.(1994).

Course Structure | Elective I | Elective II

Partial Differential Equation

Semester : IV	Group : B
Course ID : PM4/12	Full Marks : 20
Minimum number of classes required : 30	

Course Structure | Elective I | Elective II

- **Non linear first order PDE :** Complete integrals, general solution of higher order partial differential equation with constant coefficients.
- **Second order PDE :** Classification and reduction to canonical forms.
- Laplace equation, fundamental solution, properties of harmonic functions, Mean Value theorem, Greens functions.
- **Wave equation :** Elementary solution, D'Alemberts solution, solution by spherical mean, non-homogeneous equation.
- **Heat equation :** Elementary solution, fundamental solution, Mean Value formula, properties of solution.
- Solution of wave equation, Laplace equation and heat equation by separation of variables.

References

[1] I. N. Sneddon; Elements of Partial Differential Equations; McGraw-Hill, London, 1957.

[2] L. C. Evans; Partial Differential Equations; American Mathematical Society, Rhode Island, 1998.

[3] F. John; Partial Differential Equations; Narosa Publishing House, New Delhi, 1979.

Course Structure | Elective I | Elective II

Computational Mathematics (Theory)

Semester : IV	Group : A
Course ID : PM4/13	Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- An overview of theoretical computers, history of computers, overview of architecture of computer, compiler, assembler, machine language, high level language, object oriented language, programming language and importance of C programming.
- **Constants, Variables and Data type of C-Program :** Character set. Constants and variables data types, expression, assignment statements, declaration.
- **Operation and Expressions :** Arithmetic operators, relational operators, logical operators, Bitwise operators, one's complement operators, right and left shift operators, Bitwise AND operator, Bitwise OR/XOR operator, Bitwise assignment operators, conditional operators.
- **Decision Making and Branching :** decision making with if statement, if-else statement, Nesting if statement, switch statement, break and continue statement, the Goto statement.
- **Control Statements :** While statement, do-while statement, for statement.
- **Arrays :** One-dimension, two-dimension and multidimensional arrays, declaration of arrays, initialization of one and multi-dimensional arrays.
- **User-defined Functions :** Definition of functions, Scope of variables, return values and their types, function declaration, function call by value, Nesting of functions, passing of arrays to functions, Recurrence of function.
- **Pointers :** Pointer operators, address operation, pointer expression pointers and functions, pointers and array, function call by reference, dynamic memory allocations.
- **Introduction to Library functions :** stdio.h, math.h, string.h stdlib.h, time.h etc.
- **File Management :** Defining a file, opening a file, closing a file, input/output operations on file.

References

[1] B. W. Kernighan and D. M. Ritchi; The C-Programming Language; 2nd Edi.(ANSI Refresher), Prentice Hall, 1977.

[2] E. Balagurnsamy; Programming in ANSI 'C'; Tata McGraw Hill, 2004.

[3] Y. Kanetkar; Let Us C ; BPB Publication, 1999.

[4] C. Xavier; C-Language and Numerical Methods; New Age International.

[5] V. Rajaraman; Computer Oriented Numerical Methods; Prentice Hall of India, 1980.

Course Structure | Elective I | Elective II

Mathematical Logic

Semester : IV	Group : B (OP1)
Course ID : PM4/13	Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- **General Notions :** Formal language, object and meta language, general definition of a Formal Theory/Formal Logic.
- **Propositional Logic :** Formal theory for propositional calculus, derivation, proof, theorem, deduction theorem, conjunctive and disjunctive normal forms, semantics, truth tables, tautology, adequate set of connectives, applications to switching circuits, logical consequence, consistency, maximal consistency, Leindenbaum lemma, soundness and completeness theorems, algebraic semantics.
- **Predicate Logic :** First order language, symbolizing ordinary sentences into first order formulae, free and bound variables, interpretation and satisfiability, models, logical validity, formal theory for predicate calculus, theorems and derivations, deduction theorem, equivalence theorem, replacement theorem, choice rule, Prenex normal form, soundness theorem, completeness theorem, compactness theorem, First Order Theory with equality, examples of First Order Theories (groups, rings, fields etc.).

References

[1] Elliott Mendelson; Introduction to mathematical logic; Chapman & Hall; London (1997)

[2] Angelo Margaris; First order mathematical logic; Dover publications, Inc, New York (1990).

[3] S.C.Kleene; Introduction to Metamathematics; Amsterdam; Elsevier (1952).

[4] J.H.Gallier; Logic for Computer Science; John.Wiley & Sons (1987).

[5] H.B.Enderton; A mathematical introduction to logic; Academic Press; New York (1972).

Course Structure | Elective I | Elective II

Number Theory

Semester : IV Course ID : PM4/13	Group : B (OP2) Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- The Arithmetic of $\mathbb{Z}_p$, p a prime, pseudo prime and Carmichael Numbers, Fermat Numbers, Perfect Numbers, Mersenne Numbers.
- Primitive roots, the group of units $\mathbb{Z}_n^*$, the existence of primitive roots, applications of primitive roots, the algebraic structure of $\mathbb{Z}_n^*$.
- Quadratic residues and non quadratic residues, Legendre symbol, proof of the law of quadratic reciprocity, Jacobi symbols.
- Arithmetic functions, definitions and examples, perfect numbers, the Möbius Inversion formula, properties of Möbius function.
- Sum of two squares, the sum of three squares and the sum of four squares.

References

[1] Gareth A Jones and J Mary Jones; Elementary Number Theory; Springer International Edition.

[2] Neal Koblitz; A course in number theory and cryptography; Springer-Verlag, 2nd edition.

[3] D. M. Burton; Elementary Number Theory; Wm. C. Brown Publishers, Dulreque, Lowa, 1989.

[4] Kenneth. H. Rosen; Elementary Number Theory & Its Applications; AT&T Bell Laboratories, Addition-Wesley Publishing Company, 3rd Edition.

[5] Kenneth Ireland & Michael Rosen; A Classical Introduction to Modern Number Theory, 2nd edition; Springer-verlag.

[6] Richard A Mollin; Advanced Number Theory with Applications; CRC Press, A Chapman & Hall Book.

[7] Saban Alaca, Kenneth S Williams; Introduction to Algebraic Number Theory; Cambridge University Press.

Course Structure | Elective I | Elective II

Distribution Theory

Semester : IV	Group : B (OP3)
Course ID : PM4/13	Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- Topology on $C_c^\infty(\Omega)$, test functions and distributions as dual of $C_c^\infty(\Omega)$, some operations with distributions – differentiation and multiplication with smooth functions.
- Support of distribution, topology on $C^\infty(\Omega)$, compactly supported distributions as dual of $C^\infty(\Omega)$.
- Convolution of functions and distributions, fundamental solution.
- Fourier transform, Plancherel theorem, inversion theorem, Schwartz space, topology on $S(\Omega)$, tempered distributions, Fourier transforms of tempered distributions.

Further Reading :

- Sobolev spaces.
- Weak solution of PDE.

References

[1] S. Kesavan; Topics in functional analysis and applications; John Wiley & Sons, Inc., New York, 1989.

[2] Robert. S. Strichartz; A Guide to Distribution Theory and Fourier Transforms; Reprint of the 1994 original, World Scientific Publishing Co., Inc., River Edge, NJ, 2003.

[3] Walter Rudin; Functional Analysis; Second edition, International Series in Pure and Applied Mathematics, McGraw-Hill, Inc., New York, 1991.

[4] Hormander; The Analysis of Linear Partial Differential Equations I (Distribution Theory and Fourier Analysis); Reprint of the second (1990) edition, Classics in Mathematics, Springer-Verlag, Berlin, 2003.

Course Structure | Elective I | Elective II

Calculus of Variation & Integral Equation

Semester : IV	Group : B (OP4)
Course ID : PM4/13	Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- **Calculus of variations :** Variational problems with fixed boundaries, the fundamental lemma of the calculus of variations, Euler's equation, functionals dependent on several independent variables, on higher order derivatives etc.
- **Fredholm Integral Equation :** Solution by method of successive approximation, solution by method of successive substitution, direct substitution method.
- **Volterra Integral Equation :** Solution by method of successive approximation, solution by method of successive substitution, converting to initial value problem, Volterra integral equation of first kind.
- Solutions of Integral Equations with separable kernel, resolvent kernel, symmetric kernel, Hilbert-Schmidt theory.

References

[1] L. Elsgolts; Differential equations and the calculus of variations; Mir Publishers, Moscow, 1973.

[2] I. M. Gelfand & S. V. Fomin; Calculus of variations; Prentice-Hall, Englewood Cliff, New Jersey, 1963.

[3] M. L. Krasnov, G. I. Makarenko & A. I. Kiselev; Problems and exercises in the calculus of variations; Mir Publishers, Moscow, 1975.

[4] R. P. Kanwal; Linear Integral Equations; Birkhauser, Boston, 1997.

[5] A. M. Wazwaz; A First Course in Integral Equations; World Scientific, Singapore, 1997.

[6] S. G. Mikhin; Linear Integral Equations; Hindustan Book Agency, Delhi, 1960.

Course Structure | Elective I | Elective II

Automata Theory

Semester : IV	Group : B (OP5)
Course ID : PM4/13	Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- **Introduction to computability Theory :** Finite state automata, finite state mechanics, non-deterministic finite state machines, the equivalence of deterministic finite automaton and non- deterministic finite automaton, Moore machine and Mealy machines, Turing machines.
- **Language and Grammar :** Operation on Languages, Language generated by a grammar, regular Languages, context sensitive grammar, Pumping lemma and Kleene theorem.

References

[1] D. S. Malik and M. K. Sen; Discrete mathematical structure : theory and applications; Thomson, Australia, 2004.

[2] K. P. L. Mishra and N. Chandrasekaran; Theory of Computer Science; Prentice Hall of India, New Delhi, 2001.

[3] J. E. Hopcropt and J.D. Ullman; Introduction to Automata Theory, Language and computing; Norasa Publishing, New Delhi, 2000.

Course Structure | Elective I | Elective II

Mechanics

Semester : IV	Group : B (OP6)
Course ID : PM4/13	Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

Analytical Dynamics :

- Generalized coordinates, holonomic and non-holonomic systems, D' Alembert's principle and Lagrange's equations for holonomic system, energy equation for conservative fields.
- Hamiltonian variables, Hamilton canonical equations, cyclic coordinates, Routh equations, Jacobi-Poisson theorem.
- Motivating problem for calculus of variations, Euler-Lagrange's equation, shortest distance, minimum surface of revolution, Brachistochrone problem, isoperimetric problem, geodesic, fundamental lemma for calculus of variation.

References

[1] A.S. Ramsey; Dynamics Part II; Cambridge University Press, 1972.

[2] H. Goldstein; Classical Mechanics (2nd Edition); Narosa Publishing House, New Delhi.

[3] I.M. Gelfand and S.V. Fomin; Calculas of Variation; Prentice Hall of India, New Delhi.

Course Structure | Elective I | Elective II

Algebraic Geometry

Semester : IV	Group : B (OP7)
Course ID : PM4/13	Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- **Introduction :** Definition and examples.
- **Affine varieties :** Algebraic sets, Zariski topology, Hilbert's Nullstellensatz theorem, irreducibility and dimension.
- **Functions, morphisms and varieties :** Functions on affine varieties, sheaves, morphisms between affine varieties, prevarieties, varieties.
- **Projective varieties :** Projective spaces and projective varieties, cones and the projective Nullstellensatz theorem, projective varieties on a ringed spaces, the main theorem on projective varieties.
- **Dimension :** The dimension of projective varieties, the dimension of varieties, blowing up, smooth varieties.

References

[1] Robin Hartshorne; Algebraic Geometry; Springer-Verlag (1977).

[2] Joe Harris; Algebraic Geometry : a first Course; Springer-Verlag, New York (1992).

[3] William Fulton; Algebraic Curves : an Introduction to algebraic geometry; W. A. Benjamin, London (1969).

Course Structure | Elective I | Elective II

Galois Theory

Semester : IV Course ID : PM4/13	Group : B (OP8) Full Marks : 25
Minimum number of classes required : 35	

Course Structure | Elective I | Elective II

- **Finite Field :** The structure of finite field, existence of $GF(p^n)$, construction of finite fields, field of order p^n, primitive elements.
- Automorphisms of field extensions, Galois extensions, fundamental theorem of Galois theory, Galois group of a polynomial, Galois groups of quadratics, cubics and quartics.
- Solutions of polynomial equations by radicals, insolvability of general polynomial equation of order 5 by radicals.
- Roots of unity, primitive roots of unity, Cyclotomic fields, Cyclotomic polynomial, Wedderburn's theorem.
- Geometric constructions by straightedge and compass only.
- Integral extensions and Hilbert's Nullstellensatz.

References

[1] D. S. Dummit, R. M. Foote; Abstract Algebra, 2nd Edition; Wiley Student Edition.

[2] Malik, Mordeson and Sen; Fundamentals of Abstract Algebra; McGraw Hill (1997).

[3] I. N. Herstein; Topics in Algebra; Wiley Eastern Ltd. 1975.

[4] I. Stewart; Galois Theory; Chapman and Hall 1989.

[5] J. P. Escofier; Galois Theory; GTM Vol 204.Springer 2001.

[6] Joseph Rotman; Galois Theory (Second Edition); Springer, 2001.

Course Structure | Elective I | Elective II

Computational Mathematics (Practical)

Semester : IV Course ID : PM4/14/Pr	Full Marks : 25

Course Structure | Elective I | Elective II

- **Computational techniques in Some Mathematical Applications Using C-Programming :** Bisection, Trapizoidal, fixed point iteration, regular falsi, some operations on matrices, eigen values.
- Computations of some number theoretic functions such as φ, τ, σ etc.
- Some techniques for primality testing.
- Dealing with matrix and determinant.
- **Study of few UNIX commands :** cd, ls, ls -lrt, mkdir, echo, rm, rm -r, rmdir, mv, cp, cp -r, man, echo, pwd, clear, locate, grep, ifconfig, chmod.

References

[1] B. W. Kernighan and D. M. Ritchi; The C-Programming Language; 2nd Edi.(ANSI Refresher), Prentice Hall, 1977.

[2] E. Balagurnsamy; Programming in ANSI 'C'; Tata McGraw Hill, 2004.

[3] Y. Kanetkar; Let Us C; BPB Publication, 1999.

[4] C. Xavier; C-Language and Numerical Methods; New Age International.

[5] V. Rajaraman; Computer Oriented Numerical Methods; Prentice Hall of India, 1980.

Course Structure | Elective I | Elective II

Abstract Harmonic Analysis - I

Semester : III	Subject Code : 101
Course ID : PM3/E1/101	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Banach Algebra :** Normed algebra, Banach algebra, examples of Banach algebra, algebra with involution, C^*-algebra, unitization of Banach algebra, vector-valued analytic functions, resolvent set, resolvent function and its analyticity, spectrum of a point, spectral radius, ideal and maximal ideal of a Gelfand algebra, character space, maximal ideal space with Gelfand topology, Gelfand representation theorem, theory of non-unital Banach algebras.

- **Topological Group :** Basic definition and facts, subgroups, quotient groups, some special locally compact Abelian groups.

- **Measure Theory on Locally Compact Hausdörff Space :** Positive Borel measure, Riesz representation theorem, regularity properties of Borel measures, approximation by continuous functions. Complex measure, absolute continuity of measure, Radon-Nikodym theorem and its consequences. Bounded linear functionals on L^p $(1 \leq p \leq \infty)$, the dual space of $C_0(X)$ for a locally compact Hausdörff space X (the Riesz representation theorem).

- **Fourier Analysis on Euclidean Spaces :** Fourier transform on $L^1(\mathbb{R}^n)$ $(n \geq 1)$ and its various properties, inversion of Fourier transform, Fourier transform on $L^2(\mathbb{R}^n)$ $(n \geq 1)$, Plancherel theorem.

References

[1] G. B. Folland; A Course in Abstract Harmonic Analysis; CRC Press (1995).

[2] Hewitt and Ross; Abstract Harmonic analysis (Vol. I & II); Springer-Verlag (1963).

[3] M. Stein and G. Weiss; Introduction to Fourier Analysis on Euclidean Spaces; Princeton University Press (1971).

[4] Bachman and Narici; Functional Analysis; Academic Press (1966).

[5] C. E. Rickart; General Theory of Banach Algebras; D.Van Nostrand Company, Inc.

[6] G. F. Simmons; Introduction to Topology and Modern Analysis; McGraw-Hill Book Company (1963).

[7] Bachman, Narici and Beckenstein; Fourier and Wavelet Analysis; Springer.

[8] Walter Rudin; Real and Complex Analysis; McGraw-Hill Book Company (1921).

[9] Walter Rudin; Functional Analysis; Tata McGraw-Hill (1991).

[10] R. R. Goldberg; Fourier Transforms; Cambridge, N.Y. (1961).

Course Structure | Elective I | Elective II

Abstract Harmonic Analysis - II

Semester : IV Course ID : PM4/E1/101	Subject Code : 101 Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Haar Measure on Locally Compact Group :** Construction of Haar measure, properties of Haar measure, uniqueness of Haar measure (up to multiplicative constant).
- **Basic Representation Theory :** Unitary representations, Schur's lemma, representations of a group and its group algebra, Gelfand-Raikov theorem.
- **Fourier Analysis on Locally Compact Abelian Group :** The dual group, the Fourier transform, Fourier-Stieltjes transforms, positive-definite functions, Bochner's theorem, the inversion theorem, the Plancherel theorem, Pontryagin duality theorem, representations of locally compact Abelian groups, closed ideals in $L^1(G)$ for a locally compact Abelian group G, Wiener's Tauberian theorem.

References

[1] Hewitt and Ross; Abstract Harmonic analysis (Vol. I & II); Springer-Verlag (1963).

[2] Walter Rudin; Fourier Analysis on Groups; Interscience Publishers (1962).

[3] G. B. Folland; A Course in Abstract Harmonic Analysis; CRC Press (1995).

[4] Bachman and Narici; Elements of Abstract Harmonic Analysis; Academic Press, New York (1964).

[5] L. H. Loomis; An Introduction to Abstract Harmonic Analysis; D.Van Nostrand Company Inc. (1953).

[6] Y. Katznelson; An Introduction to Harmonic Analysis; Dover Publications, Inc. (1976).

Course Structure | Elective I | Elective II

Algebraic Aspects of Cryptology - I

Semester : III Course ID : PM3/E1/102	Subject Code : 102 Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

Theory (Full Marks : 40) :

- **Probability Theory :** Basic laws, Bernoulli and binomial random variables, the geometric distribution, Markov's inequality, Chebyshev's inequality, Chernoff's bound.
- **Basic Algorithmic Number Theory :** Faster integer multiplication, extended Euclid's algorithm, quadratic residues, Legendre symbols, Jacobi symbols, Chinese Remainder theorem, fast modular exponentiation, choosing a random group element, finding a generator of a cyclic group, finding square roots modulo a prime p, polynomial arithmetic, arithmetic in finite fields, factoring polynomials over finite fields, isomorphisms between finite fields, computing order of an element, computing primitive roots, fast evaluation of polynomials at multiple points, primality testing, Miller-Rabin Test, Generating random primes, primality certificates, algorithms for factorizing, algorithm for computing discrete logarithms.
- Complexity analysis of various number theoretic algorithms.
- **Public Key Cryptography and allied applications :** DLP, Diffie-Hellman key exchange, RSA, El-Gamal, Rabin. Public key based signature schemes, Oblivious transfer protocols.

Further Reading :

- **Complexity Theory :** P, NP, P vs NP question, polynomial time reductions (emphasis on oracle machines), NP-Complete problems, randomized algorithms, probabilistic polynomial time, non-uniform polynomial time.
- **Algebraic Geometry :** Affine Algebraic Sets, parametrizations of affine varieties, ordering of the monomials in $K[X_1, X_2, \ldots, X_n]$, a division algorithm in $K[X_1, X_2, \ldots, X_n]$, Monomial ideals and Dickson's Lemma, Hilbert Basis Theorem, Gröbner basis, properties, Buchberger's Algorithm.

Practical (Full Marks : 10) :

- C implementation of various primitives for cryptographic schemes.

References

[1] Steven D. Galbraith; Mathematics of Public Key Cryptography; Cambridge university press.

[2] D. R. Stinson, Cryptography; Theory & Practice; CRC Press Company, 2002.

[3] Jeffery Hoffstein, Jill Pipher, J.H.Silverman; An Introduction to Mathematical Cryptography; Springer.

[4] Jonathan Katz, Yehuda Lindell; Introduction to Modern Cryptography; Chapman & Hall/CRC.

[5] Neal Koblitz; A course in number theory and cryptography; Springer-Verlag, 2nd edition.

[6] D. M. Burton; Elementary Number Theory; Wm. C. Brown Publishers, Dulreque, Lowa, 1989.

[7] Kenneth. H. Rosen; Elementary Number Theory & Its Applications; AT&T Bell Laboratories, Addition-Wesley Publishing Company, 3rd Edition.

[8] Kenneth Ireland & Michael Rosen; A Classical Introduction to Modern Number Theory, 2nd edition; Springer-verlag.

[9] Richard A Mollin; Advanced Number Theory with Applications; CRC Press, A Chapman & Hall Book.

[10] Saban Alaca, Kenneth S Williams; Introduction to Algebraic Number Theory; Cambridge University Press.

[11] Jay R Goldman; The Queen of Mathematics : a historically motivated guide to number theory; A K Peters Ltd.

Course Structure | Elective I | Elective II

Algebraic Aspects of Cryptology - II

Semester : IV Course ID : PM4/E1/102	Subject Code : 102 Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

Theory (Full Marks : 40) :

- **Private Key Cryptography :** Private key encryption, perfectly secure encryption and its limitations, semantic security, pseudo-random number generator.
- **Stream Cipher :** Boolean function, LFSR, non-linear combiner model, linear complexity, Walsh transformation, Hadamard matrix, Correlation immunity, attacks on Boolean functions, S-Box, Some stream ciphers such as RC4, Attack on RC4.
- **Block Cipher :** Data Encryption Standard, Modes of Operations,The Advanced Encryption Standard, Basic Algorithms, Decryptions.
- **Hash functions :** Security properties of Hash functions, birthday attack, MAC, Construction of Hash functions, Number theoretic hash functions, Merkle-Damgard construction.
- **Computational approach to cryptography :** Basic ideas of computational security, efficient algorithms and negligible success probability, proof by reduction, security notions : CPA, CCA, CCA2, Security for multiple encryptions.
- **More PKCs :** Goldwasser-Micali, Paillier.
- **Secret Sharing Schemes :** Shamir's Secret Sharing Scheme, more on Secret Sharing schemes such as cheating immune, cheating identifiable etc, visual cryptography, DNA secret sharing scheme.

Further Reading :

- **Elliptic curves :** Properties of elliptic curves, elliptic curve over real and modulo a prime, torsion points, secret sharing scheme based on elliptic curve.
- **Lattices :** Basic notions, Hermite and Minkowski's bounds, computational problems in Lattices, LLL-reduced basis, the LLL Algorithm, Babai's Nearest Plane Algorithm, low exponent attack on RSA using lattices, GGH, NTRU.

Practical (Full Marks : 10) :

- Sage implementation of various primitives for cryptographic schemes.

References

[1] Steven D. Galbraith; Mathematics of Public Key Cryptography; Cambridge university press.

[2] D. R. Stinson, Cryptography; Theory & Practice; CRC Press Company, 2002.

[3] Jeffery Hoffstein, Jill Pipher, J.H.Silverman; An Introduction to Mathematical Cryptography; Springer.

[4] Jonathan Katz, Yehuda Lindell; Introduction to Modern Cryptography; Chapman & Hall/CRC.

[5] Neal Koblitz; A course in number theory and cryptography; Springer-Verlag, 2nd edition.

[6] D. M. Burton; Elementary Number Theory; Wm. C. Brown Publishers, Dulreque, Lowa, 1989.

[7] Kenneth. H. Rosen; Elementary Number Theory & Its Applications; AT&T Bell Laboratories, Addition-Wesley Publishing Company, 3rd Edition.

[8] Kenneth Ireland & Michael Rosen; A Classical Introduction to Modern Number Theory, 2nd edition; Springer-verlag.

[9] Richard A Mollin; Advanced Number Theory with Applications; CRC Press, A Chapman & Hall Book.

[10] Saban Alaca, Kenneth S Williams; Introduction to Algebraic Number Theory; Cambridge University Press.

[11] Jay R Goldman; The Queen of Mathematics : a historically motivated guide to number theory; A K Peters Ltd.

Course Structure | Elective I | Elective II

Advanced Real Analysis - I

Semester : III	Subject Code : 103
Course ID : PM3/E1/103	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Ordinal numbers :** Order types, well-ordered sets, transfinite induction, ordinal numbers, comparability of ordinal numbers, arithmetic of ordinal numbers, first uncountable ordinal Ω.
- **Descriptive properties of sets :** Perfect sets, decomposition of a closed set in terms of perfect sets of first category, 2nd category and residual sets, characterization of a residual set in a compete metric space, Borel sets of class α, ordinal $\alpha < \Omega$. Density point of a set in $\mathbb{R}$, Lebesgue density theorem.
- **Functions of some special classes :** Borel measurable functions of class $\alpha\,(\alpha < \Omega)$ and its basic properties, comparison of Baire and Borel functions, Darboux functions of Baire class one.
- **Continuity :** The nature of the sets of points of discontinuity of Baire one functions, approximate continuity and its fundamental properties, characterization of approximate continuous functions.
- **Henstock integration on the real line :** Concepts of δ-fine partition of the closed interval $[a, b]$ where δ is a positive function on $[a, b]$, Cousin's lemma, definition of Henstock integral of a function over the interval $[a, b]$ and its basic properties. Saks-Henstock lemmas and its applications, continuity of the indefinite integral, fundamental theorem, convergence theorems, absolute Henstock integrability, characterization of Lebesgue integral by absolute Henstock integral.

References

[1] A.M.Bruckner, J.B.Bruckner & B.S.Thomson; Real Analysis; Prentice-Hall, N.Y.1997.

[2] I.P.Natanson; Theory of Functions of Real Variable, Vol.I & II; Frederic Ungar Publishing 1955.

[3] C.Goffman; Real Functions; Rinehart Company, N.Y, 1953.

[4] P.Y.Lee; Lanzhou Lectures on Henstock Integration; World Scientific Press, 1989.

[5] J.F. Randolph; Basic Real and Abstract Analysis; Academic Press, N.Y, 1968.

[6] S.M.Srivastava; A Course on Borel Sets; Springer, N.Y, 1998.

Course Structure | Elective I | Elective II

Advanced Real Analysis - II

Semester : IV	Subject Code : 103
Course ID : PM4/E1/103	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Derivative :** Banach-Zarecki theorem, derivative and integrability of absolutely continuous functions, Lebesgue point of a function, determining a function by its derivative.
- **General Measure and Integration :** Additive set functions, measure and signed measures, limit theorems, Jordan and Hahn decomposition theorems, complete measures, integrals of non-negative functions, integrable functions, absolute continuous and singular measures, Radon-Nikodym theorem, Radon-Nikodym-derivative in a measure space.
- **Fourier Series :** Fourier series of functions of class L, Fejer-Lebesgue theorem, integration of Fourier series, Cantor-Lebesgue theorem on trigonometric series, Riemann's theorem on trigonometric series, uniqueness of trigonometric series.
- **Distribution Theory :** Test functions, compact support functions, distributions, operation on distributions, local properties of distributions, convergence of distributions, differentiation of distributions and some examples, derivative of locally integrable functions, distribution of compact support, direct product of distributions and its properties, convolution and properties of convolutions.

References

[1] A.M.Bruckner, J.B.Bruckner & B.S.Thomson; Real Analysis; Prentice-Hall, N.Y.1997.

[2] R.L.Jeffery; The Theory of Functions of a Real Variable; Toronto University Press,1953.

[3] I.P.Natanson; Theory of Functions of Real Variable, Vol.I & II; Frederic Ungar Publishing 1955.

[4] F.G.Friedlander; Introduction to the Theory of Distributions; Cambridge Univ Press,1982

[5] H.L.Royden; Real Analysis; Macmillan, N.Y, 1963.

[6] S. Kesavan; Topics in Functional Analysis and its Applications; Wiley Eastern Ltd, New Delhi, 1989.

Course Structure | Elective I | Elective II

Advanced Complex Analysis - I

Semester : III	Subject Code : 104
Course ID : PM3/E1/104	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Analytic Functions :** Convex function, Mean values, the function $A(r)$, Borel-Caratheodory's theorem.
- **Entire function :** Entire functions, entire transcendental functions, order and type of an entire functions, distributions of zeros of analytic functions, the function $n(r)$, Jensen's theorem and Jensen's inequality. Order and type in terms of Taylor coefficients. Infinite product of complex number and complex functions, Weierstrass's factorization theorem, Hadamard's factorization theorem
- **Analytic Continuations :** Direct analytic continuations, uniqueness of analytic continuation along a curve, Monodromy theorem and its consequence, analytic continuation via Reflection.
- **Harmonic Functions :** Basic properties of harmonic functions, harmonic function on a disk, subharmonic and superharmonic functions, Dirichlet problems for a disk, Green's function.

References

[1] L. V. Ahlfors; Complex Analysis; McGraw Hill, 1979.

[2] R. V. Churchill & J. W. Brown; Complex variables and applications; McGraw Hill.

[3] J. B. Conway; Functions of one complex variable; Springer-Verlag, Int. student edition, Narosa Publishing House, 1980.

[4] T.W. Gamelin; Complex Analysis; Springer International Edition, 2001.

[5] A. S. B. Holland; Theory of entire functions; Academic Press, 1973.

[6] S. Lang; Complex Analysis; Forth edition, Springer-Verlag,1999.

[7] I. Marcushevich; Theory of functions of a complex variable Vol-I,II,III; Prentice- -Hall,1965.

[8] S. Ponnusamy; Foundations of complex analysis; Narosa Publishing House, 1997.

[9] H. A. Priestly; Introduction to complex analysis; Clarendon Press, Oxford, 1990.

[10] R. Remmert; Theory of Complex Functions; Springer Verlag, 1991.

[11] A.R. Shastri; An Introduction to Complex Analysis; Macmilan India, New Delhi, 1999.

[12] E. C. Tichmarsh; Theory of functions; Oxford University Press, London, 1939.

Course Structure | Elective I | Elective II

Advanced Complex Analysis - II

Semester : IV	Subject Code : 104
Course ID : PM4/E1/104	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Meromorphic Functions :** Some basic properties of meromorphic functions, Mittag-Leffler's theorem, application of Mittag-Leffler's theorem for simple poles, Gamma functions and its properties, Riemann zeta functions, Riemann's functional equations, Runge's theorem.
- **The Range of Analytic Functions :** Bloch's theorem, The little Picard theorem, Schottky's theorem, The Great Picard theorem.
- **Inverse and Implicit Functions of Complex Variables :** Inverse functions — the single valued case, the multivalued case, examples of Lagrange's series, functions of two variables, Weierstrass Preparation theorem, the Implicit function theorem.
- **Univalent Functions :** Basic properties of univalent functions, necessary and sufficient conditions for a function to be univalent, Area theorem, Distortions theorem.

References

[1] L. V. Ahlfors; Complex Analysis; McGraw Hill, 1979.

[2] R. V. Churchill & J. W. Brown; Complex variables and applications; McGraw Hill.

[3] J. B. Conway; Functions of one complex variable; Springer-Verlag, Int. student edition, Narosa Publishing House, 1980.

[4] T.W. Gamelin; Complex Analysis; Springer International Edition, 2001.

[5] A. S. B. Holland; Theory of entire functions; Academic Press, 1973.

[6] S. Lang; Complex Analysis; Forth edition, Springer-Verlag,1999.

[7] I. Marcushevich; Theory of functions of a complex variable Vol-I,II,III; Prentice- -Hall,1965.

[8] S. Ponnusamy; Foundations of complex analysis; Narosa Publishing House, 1997.

[9] H. A. Priestly; Introduction to complex analysis; Clarendon Press, Oxford, 1990.

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[11] A.R. Shastri; An Introduction to Complex Analysis; Macmilan India, New Delhi, 1999.

[12] E. C. Tichmarsh; Theory of functions; Oxford University Press, London, 1939.

Course Structure | Elective I | Elective II

Advanced Riemannian Manifold - I

Semester : III	Subject Code : 105
Course ID : PM3/E1/105	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Connections :** Affine Connections (Koszul), Torsion and Curvature tensor field on Affine Connection, Covariant Differential. Parallel Transport.
- **Riemannian Manifold :** Riemannian manifolds and sub manifolds, Riemannian Connection, Riemann Curvature tensor. The Exponential Map, Normal Neighborhoods and Normal Coordinates, Geodesics of the Model Spaces.
- The Jacobi Equation, Computations of Jacobi Fields, Conjugate Points, The Second Variation Formula.
- **Bundle Theory :** Principal Fibre Bundles, Linear Frame Bundle, Associated Fibre Bundle, Induced Bundle, Bundle Homomorphism, Linear Connection, Lift of a Vector Field.

References

[1] Kobayashi & Nomizu; Foundations of Differential Geometry; Vol-I, Interscience Publishers, 1963.

[2] W.M.Boothby; An Introduction to Differentiable Manifolds and Riemanian Geometry, Academic Press, Revised, 2003.

[3] Eisenhart, L.P; Riemannian Geometry; Princeton University Press, 1949.

[4] John M. Lee; Riemannian manifolds : An Introduction to curvature; Springer.

[5] Kumaresan, S; A course in Differential Geometry and Lie-groups; Hindustan Book Agency.

Course Structure | Elective I | Elective II

Advanced Riemannian Manifold- II

Semester : IV	Subject Code : 105
Course ID : PM4/E1/105	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Special Theory of Relativity :** Inertial frame of reference, Principles of special theory of relativity, special Lorentz transformations, Minkowski space time, Causality, Time dilation and Fitzgerald contraction, Velocity 4-vector and acceleration 4-vector in a Minkowski space-time, Lorentz transformation law of velocity 4-vector, Mass and Momentum, World-line of a particle in a Minkowski space-time.
- **General Theory of Relativity :** Principles of general theory of relativity, Energy Momentum tensor, Perfect Fluid, Einstein Field equation, Acceleration of a particle in a weak gravitational field, Schwarzchild Metric, Einstein Universe, De-Sitter's Universe, Manifold of General Theory of Relativity.

References

[1] Eric A.Lord; Tensors, Relativity and Cosmology; Tata McGraw-Hill Pub. New Delhi.

[2] S.K.Bose; An Introduction to General Relativity; Wiley Eastern Ltd. 1985.

[3] S.Weinberg; Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity; John Wiley & Sons. Inc. 1972.

[4] Barrett O' Neill; Semi-Riemannian Geometry with applications to Relativity; Academic Press, 1983.

Course Structure | Elective I | Elective II

Advanced Algebraic Topology - I

Semester : III Course ID : PM3/E1/106	Subject Code : 106 Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Cellular homology of a CW complex, Isomorphism between singular and cellular homology of a CW-complex; Homology with arbitrary coefficient group and Universal coefficient theorem.
- The product of CW-complexes and the tensor product of chain complexes; Singular chain complex of a product space; Homology of the tensor product of the product space (Kunneth theorem); Eilenberg-Zilber Theorem.
- Cohomology groups – basic properties, Universal coefficient theorem for cohomology, geometric interpretation of co-cycles and co-chains; excision property and Mayer-Vietoris sequence.

Further Reading :

- Generalized Jordan curve Theorem.
- de Rham's Cohomology and its relation with singular homology; de Rham's Theorem.

References

[1] G.E.Bredon; Topology and Geometry; Springer-Verlag GTM 139 (1993).

[2] A.Dold; Lectures on Algebraic Topology; Springer-Verlag (1980).

[3] W.Fulton; Algebraic Topology, A First Course; Springer-Verlag (1995).

[4] M. J. Greenberg and J. R. Harper; Algebraic Topology : A first course; Perseus Books (Mathematical Lecture Notes series) (1981).

[5] A.Hatcher; Algebraic Topology; Cambridge University Press (2002).

[6] William S.Massey; A Basic Course in Algebraic Topology; Springer-Verlag, New York Inc.(1993).

[7] C.R.F.Maunder; Algebraic Topology; Dover Pub. N.Y. (1996).

[8] J.J.Rotman; An Introduction to Algebraic Topology; Springer-Verlag, N.Y. (1988).

[9] H.Schubert; Topology; Macdonald Technical and Scientific, London (1964).

[10] James W. Vick; Homology Theory : An introduction to Algebraic Topology; Springer-Verlag, N. Y.(1994).

Course Structure | Elective I | Elective II

Advanced Algebraic Topology - II

Semester : IV	Subject Code : 106
Course ID : PM4/E1/106	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Cohomology Theory (continued) :** Cross product and Kunneth formula, Cup and cap product; Orientations, Poincare duality and other duality theorems.
- The higher homotopy groups - basic constructions, properties and examples; Homotopy groups of spheres, Whitehead's theorem, classification of vector bundles and fibre bundles; Cellular approximation and CW approximation. Excision and homotopy groups, Hurewicz theorem, Eilenberg-MacLane space. Homotopy construction of cohomology, Fibrations.

Further Reading :

- The fundamental group and covering spaces of a Graph;
- Euler characteristics of a finite graph; covering spaces of a graph.
- Obstruction Theory.

References

[1] G.E.Bredon; Topology and Geometry; Springer-Verlag GTM 139 (1993).

[2] A.Dold; Lectures on Algebraic Topology; Springer-Verlag (1980).

[3] W.Fulton; Algebraic Topology, A First Course; Springer-Verlag (1995).

[4] M. J. Greenberg and J. R. Harper; Algebraic Topology : A first course; Perseus Books (Mathematical Lecture Notes series) (1981).

[5] A.Hatcher; Algebraic Topology; Cambridge University Press (2002).

[6] William S.Massey; A Basic Course in Algebraic Topology; Springer-Verlag, New York Inc.(1993).

[7] C.R.F.Maunder; Algebraic Topology; Dover Pub. N.Y. (1996).

[8] J.J.Rotman; An Introduction to Algebraic Topology; Springer-Verlag, N.Y. (1988).

[9] H.Schubert; Topology; Macdonald Technical and Scientific, London (1964).

[10] James W. Vick; Homology Theory : An introduction to Algebraic Topology; Springer-Verlag, N. Y.(1994).

Course Structure | Elective I | Elective II

Universal Algebra, Category Theory & Lattice Theory - I

Semester : III	Subject Code : 107
Course ID : PM3/E1/107	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Universal algebra, examples, subalgebra, subuniverse, congruences and quotient algebra. Homomorphisms of universal algebra, kernel of a homomorphism. First Isomorphism Theorem, Second Isomorphism Theorem, Third Isomorphism Theorem. Semigroup of endormorphisms of a universal Algebra. Birkhoff's theorem. Special faithful representation of universal algebras into semigroups. Theorem of Cohn-Rebane. Free algebra. Direct and subdirect product of universal algebras. Subdirectly irreducible universal algebras. Class operators. Turski's theorem. Identities. Term algebra, universal mapping property, k-free universal algebra. Variety, Birkhoff's theorem on variety. Macev's conditions.

References

[1] Stanley Burries and H. P. Sankappanavan; Universal Algebra; Springer-Verlag.

Course Structure | Elective I | Elective II

Universal Algebra, Category Theory & Lattice Theory - II

Semester : IV	Subject Code : 107
Course ID : PM4/E1/107	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Category, examples. Dual category, special morphisms. Monic and epic. Retraction and coretraction. Functor, forgetful functor, faithful functor. Product category, bifunctor, natural transformation, representable functor, embedding. Yoneda's lemma and its applications. Adjoint functor. Initial object, terminal object, zero object. Limit and colimit. Pull back diagram and push out diagram.
- Lattice, sublattice, generators of a sublattice, ideal, dual ideal. Homomorphisms and congruences. Distributive lattice. Characterization theorems and representation theorems on distributive lattice. Structure of the lattice of congruences of lattices. Semi modular lattices. Characterization theorems and representation theorems. Stone's representation theorem on Boolean Algebra. Algebraic lattice – characterization theorems and representation theorems. One-to-one correspondence between ideals and congruences of Boolean Algebra and generalized Boolean Algebra.

References

[1] S. Maclane; Category theory for working mathematician; Springer-Verlag.

[2] B. Mitchel; Theory of categories; Academic Press (1969).

[3] Gratzer; Lattice theory; Verlag, Basel.

[4] Birkhoff; Lattice theory; AMS.

Course Structure | Elective I | Elective II

Advanced Graph Theory - I

Semester : III	Subject Code : 108
Course ID : PM3/E1/108	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Degree sequence :** Havel- Hakimi theorem, Statement of Erdos and Gallai theorem. Harary graphs.
- **Connectivity :** Cut vertices, non separable graphs, Blocks. Vertex and edge connectivity, Relationship of connectivities and minimum degree, Menger's theorem.
- **Eulerian graphs and Hamiltonian graphs :** Ore's Theoren, Dirac' theorem, Closure of a Graph, Bondy and Chavtal theorem. Uniqueness of closure, Necessary and sufficient condition for a graph to be Hamiltonian.
- Distance, centre, radius and diameter of a graph, Cartesian product of graphs. Cut set and its properties.
- **Planar graph :** Kuratowski's theorem, Wagner theorem. Embedding on a Torus, The genus of a graph. Dual of a planar graph, combinatorial dual. Outerplanar graphs. Its forbidden subgraph characterization, maximal outer planar graphs.
- **Colouring :** vertex colouring, chromatic number, k- critical graphs, Mycielski construction, Greedy colouring algorithm, Brooks theorem, chromatic polynomial, Map colouring, Five colour theorem, The Four colour theorem(statement and brief history). Applications of graph Colouring.

References

[1] J. A. Bondy and U. S. R. Murty; Graph theory with application; Academic Press; 1979.

[2] G. Chartrand, L. Lesniak and P. Zhang; Graphs and Digraphs; Chapman and Hall, 2011.

[3] D. B. West; Introduction to Graph Theory; Prentice Hall of India, New Delhi; 2012.

[4] Robin J. Wilson; Graph Theory; Pearson Education Asia; 2002.

[5] R. Diestel; Graph Theory; Springer-Verlag, Berlin, 2005.

Course Structure | Elective I | Elective II

Advanced Graph Theory - II

Semester : IV Course ID : PM4/E1/108	Subject Code : 108 Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Directed graphs :** Directed graphs and binary relation, directed walks, trails, paths and connectedness, Euler digraphs, Characterization of Euler digraph. Acyclic digraphs and decyclization. Tournaments and their properties, Strong tournament. Topological sorting of the vertices of a tournament and for general acyclic digraphs.
- **Power of Graphs and Line graph :** Hamiltonian-connected graphs, the square and cube of a graph. Definition and characterization of line graphs. Forbidden subgraphs of line graphs. The total graph of a graph.
- **Edge colouring :** Edge colouring and chromatic index, Konigs theorem and chromatic index of a complete graph.
- **Clique and Stable set :** Clique number, clique cover number and stability number of a graph. Definition of a Perfect graph. Perfect graph Theorem (Proof not required).
- **Triangulated graphs :** Characterization of triangulated graphs with perfect scheme and minimal vertex separator. Transitive orientation and comparability graphs.
- **Interval Graphs :** Intersection graph, definition and characterizations of an interval graph. Some application of interval graphs.
- **Ramsey Theory :** Ramsey theorem, A party problem, Definition and some examples of Ramsey numbers, Ramsey Graph.

References

[1] G. Chartrand, L. Lesniak and P. Zhang; Graphs and Digraphs; Chapman and Hall, 2011.

[2] D. B. West; Introduction to Graph Theory; Prentice Hall of India, New Delhi; 2012.

[3] R. Diestel; Graph Theory; Springer-Verlag, Berlin, 2005.

[4] M. C. Golumbic; Algorithmic Graph Theory and Perfect Graph; Elsevier, 2004.

Course Structure | Elective I | Elective II

Algebraic Coding Theory - I

Semester : III Course ID : PM3/E1/109	Subject Code : 109 Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- The Communication channel. The Coding Problem. Types of Codes. Block Codes. Error-Detecting and Error-Correcting Codes. Linear Codes. The Hamming Metric. Description of Linear Block Codes by Matrices. Dual Codes. Standard Array. Syndrome. Step-by-step Decoding Modular Representation. Error-Correction Capabilities of Liner Codes. Bounds on Minimum Distance for Blcok Codes. Plotkin Bound. Hamming Sphere packing Bound. Varshamov-Gilbert-Sacks Bound. Bounds for Burst-Error Detecting and Correcting Codes. Important Linear Block Codes. Hamming Codes. Golay Codes. Perfect Codes. Quasi—perfect Codes. Reed-Muller Codes. Codes derived from Hadamard Matrices. Product Codes Concatenated Codes. Tree Codes. Convolutional Codes. Description of Linear Tree and Convolutional Codes by Matrics. Standard Array. Bounds on Minimum distance for Convolutional Codes. V.G.S., bound. Bounds for Burst-Error Detecting and Correcting Convolutional Codes. The Lee metric, packing bound for Hamming code w.r.t. Lee metric.

References

[1] Steven Roman; Coding and Information Theory; Springer-Verlag.

[2] Richard Hamming; Coding and Information Theory; Prentice Hall.

[3] F.J. MacWilliams, N.J.A. Sloane; The Theory of Error-Correcting Codes; (North-Holland Mathematical Library).

[4] Norman L Biggs; Codes- An Introduction to Information Communication and Cryptography; Springer Undergraduate Mathematics Series.

Course Structure | Elective I | Elective II

Algebraic Coding Theory - II

Semester : IV	Subject Code : 109
Course ID : PM4/E1/109	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- The Algebra of polynomial residue classes. Galois Fields. Multiplicative group of a Galois field. Cyclic Codes. Cyclic Codes as Ideals. Matrix Description of Cyclic Codes. Hamming and Golay Codes as Cyclic Codes. Error Detection with cyclic Codes. Error-Connection procedure for Short Cyclic Codes. Shortened Cyclic Codes. Pseudo Cyclic Codes. Code symmetry. Invariance of Codes under transitive group of permutations. **Bose-Chaudhuri-Hocquenghem** (BCH) Codes. Majority-Logic Decoding. BCH bounds. Reed-Solomon (RS) Codes. Majority-Logic Decodable Codes. Majority-Logic Decoding. Singleton bound. The Griesmer bound. Maximum-distance Separable (MDS) Codes. Generator and Parity-check matrics of MDS Code. Weight Distribution of MDS Code. Necessary and Sufficient conditions for linear code to be an MDS Code. MDS codes from RS codes. Abramson Codes. Close-loop burst-Error correcting codes (Fire codes). Error Locating Codes.

References

[1] Steven Roman; Coding and Information Theory; Springer-Verlag.

[2] Richard Hamming; Coding and Information Theory; Prentice Hall.

[3] F.J. MacWilliams, N.J.A. Sloane; The Theory of Error-Correcting Codes; (North-Holland Mathematical Library).

[4] Norman L Biggs; Codes- An Introduction to Information Communication and Cryptography; Springer Undergraduate Mathematics Series.

Course Structure | Elective I | Elective II

Differential Topology - I

Semester : III	Subject Code : 110
Course ID : PM3/E1/110	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Manifolds and smooth maps. Immersions; subimmersions; level surfaces; transversal maps; Sard's theorem. Morse functions. Embedding manifolds in Euclidean spaces.
- Manifolds with boundary. Brouwer Fixed point theorem. Genericity of transversal maps. Tabular neighbourhood theorem. Transversality homopoty theorem.
- Intersection theory mod 2. Winding number and the Jordan Brower separation theorem. The Borsuk-Ulam theorem.

References

[1] V. Guillemin and A. Pollack; Differential Topology; Prentice-Hall, Innc. Englewood Cliffs, New Jersey (1974).

[2] M. Hirsch; Differential Topology; Graduate Texts in Mathematics Series, Springer-Verlag (1976).

[3] A. A. Kosinski; Differential Manifolds; Academic Press (1993).

Course Structure | Elective I | Elective II

Differential Topology - II

Semester : IV	Subject Code : 110
Course ID : PM4/E1/110	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Orientation on a manifold. Oriented intersection number, The Fundamental theorem of Algebra. Euler characteristic. Lefschetz Fixed point theorem. Homotopy invariance of Lefschetz number. Splitting proposition. Local computation of the Lefschetz number. Index of vector fields on a manifold. Poincare-Hopf index theorem. The Hopf degree theorem. Isotopy lemma. The Euler characterization and Triangulations. Integration on manifolds. Exterior algebra. Differential Forms. Exterior derivative. Stoke's theorem. Integration and mappings. Degree formula. Gauss map. Gauss-Bonnet theorem.

References

[1] V. Guillemin and A. Pollack; Differential Topology; Prentice-Hall, Innc. Englewood Cliffs, New Jersey (1974).

[2] M. Hirsch; Differential Topology; Graduate Texts in Mathematics Series, Springer-Verlag (1976).

[3] A. A. Kosinski; Differential Manifolds; Academic Press (1993).

Course Structure | Elective I | Elective II

Dynamical System and Integral Equations - I

Semester : III	Subject Code : 111
Course ID : PM3/E1/111	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Linear Systems :** Uncoupled linear systems, Diagonalization, exponentials of operators, The fundamental theorem for linear systems, Linear systems in $\mathbb{R}^2$, Complex eigenvalues, Multiple eigenvalues.
- **Nonlinear Systems :** Local theory, Some preliminary concepts and definitions, The fundamental existence-uniqueness theorem, Dependence on initial conditions and parameters, The maximal interval of existence, The flow defined by a Differential Equation, Linearization, The Stable-Manifold theorem, The Hartman-Grobman theorem, Stability and Liapunov functions, Saddles, nodes, Foci and Centre.
- **Nonlinear System :** Global theory, Dynamical Systems and Global Existence Theorems, Limit sets and attractors, periodic orbits, Limit cycles and separatrix cycles, The Poincaré map, The stable manifold theorem for periodic orbits.
- **Nonlinear Systems :** Bifurcation theory, Structural Stability and Peixoto's theorem, Bifurcation at Non-hyperbolic Equilibrium Points, Hopf bifurcation and bifurcations of limit cycles from a multiple focus.

References

[1] Perko L; Differential Equations and Dynamical Systems; Springer.

[2] Nemytskii, V.V. and Stepanov, V.V; Qualitative Theory of Differential Equations; Princeton University Press, Princeton.

Course Structure | Elective I | Elective II

Dynamical System and Integral Equations - II

Semester : IV	Subject Code : 111
Course ID : PM4/E1/111	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Symmetric Kernels :** Orthonormal system of functions. Fundamental properties of eigenvalues and eigenvalues and eigenfunctions for symmetric kernels. Expansion in eigenfunction and bilinear form. Hilbert Schmidt theorem and some immediate consequences. Solution of integral equations with symmetric kernels.
- **Green's Functions :** Approach to reduce BVP of a self-adjoint DE with homogeneous boundary conditions to integral equation forms. Auxiliary problem with more general and inhomogeneous boundary conditions. Modified Green's function.
- **Integral Transforms :** Fourier Transforms : Fourier's integral theorem, Fourier transforms, Fourier sine and cosine transforms, Fourier transform of derivatives, The calculation of the Fourier transform of some simple functions, The Fourier transforms of rational functions, the convolution integral, Parseval's theorem for cosine and sine transforms, The solution of integral equations of convolution type.
- **Laplace Transform :** Calculation of the Laplace transform of some elementary functions, Rules of manipulation of the Laplace transform, Laplace transform of derivatives, Relations involving integrals,The convolution of two functions, The inversion formula for the Laplace transform, The solution of ODE : Initial value problems for a linear equation with constant coefficients, Linear Differential equations with variable coefficients, Solution of Integral Equations.

References

[1] R.P.Kanwal; Linear Integral Equations : Theory and Technique; Academic Press Inc.

[2] W.V.Lovitt; Linear Integral Equations; Dover Publications, N.Y. 1950.

[3] W.Pogovzelski; Integral equations and their applications, Vol-I; Pergamon Press, Oxford, 1966.

[4] F.G.Tricomi; Integral Equations; Wiley, N.Y. 1957.

[5] I.N.Sneddon; The use of Integral transforms; TMH Edition.

[6] L.Debnath; Integral transforms and their applications; CRC Press 1995.

[7] E.T.Whittacker and G.N.Watson; A Course of Modern Analysis; Cambridge University Press.

Course Structure | Elective I | Elective II

Modules and Rings - I

Semester : III	Subject Code : 201
Course ID : PM3/E2/201	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Morphisms. Exact sequences. The three lemma. The four lemma. The Five lemma. Theorem (Butterfly of Zausenhauss). Product and co-product of R-modules. Free modules.
- Noetherian module and Artinian module, composition series.
- Projective modules. Injective modules. Direct seem of projective modules. Direct product of injective modules.
- Divisible groups. Embedding of a module in an injective module. Tensor product of modules. Noetherian and Artinian modules. Finitely generated modules. Jordan-Hölder theorem. Indecomposable modules. Krull-Schmidt theorem. Semi-simple modules. Submodules, homomorphic images and direct sum of semi-simple modules.

References

[1] T.S.Blyth; Module Theory; Clarendon Press, London.

[2] T.Y.Lam; Noncommutative Rings; Springer-Verlag, 1991.

[3] I.N.Herstein; Noncommutative Rings; C. Monographs of AMS, 1968.

[4] T.W. Hungerford; Algebras; Springer, 1980.

Course Structure | Elective I | Elective II

Modules and Rings - II

Semester : IV	Subject Code : 201
Course ID : PM4/E2/201	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Prime ideals, m-system, prime radical of an ideal, prime radical of a ring. Semiprime ideal, n-system, prime rings, semiprime ring as a subdirect product of prime ring, prime ideals and prime radical of matrix ring.
- Subdirect sum of rings, representation of a ring as a subdirect sum of rings. Subdirectly irreducible ring, Birkhoff theorem on subdirectly irreducible ring. Subdirectly irreducible Boolean ring.
- Local ring, characterizations of local ring, local ring of formal power series.
- Semisimple module, semisimple ring, characterizations of semisimple module and semisimple ring, Wedderburn-Artin theorem on semisimple ring.
- Simple ring, characterization of Artinian simple ring.
- The Jacobson radical, Jacobson radical of matrix ring, Jacobson semisimple ring, relation between Jacobson semisimple ring and semisimple ring, Hopkins-Levitzki theorem, Nakayama's lemma, regular ring, relation among semisimple ring, regular ring and Jacobson semisimple ring. Primitive ring, structure of primitive ring, Jacobson-Chevalley density theorem, Wedderburn-Artin theorem on primitive ring.
- Lower nil radical, upper nil radical, nil radical, Brauer's lemma, Kothe's conjecture, Levitzki theorem.

References

[1] T.Y.Lam; Noncommutative Rings; Springer-Verlag.

[2] I.N.Herstein; Noncommutative rings; Carus monographs of AMS, 1968.

[3] N. Jacobson; Structure of Rings; AMS.

[4] L.H. Rowen; Ring theory (student edition); Academic Press, 1991.

[5] T.W. Hungerford; Algebra; Springer, 1980.

Course Structure | Elective I | Elective II

Advanced Functional Analysis - I

Semester : III	Subject Code : 202
Course ID : PM3/E2/202	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Balanced, convex and absorbing sets and their properties, definitions and examples of linear topological space (l.t.s.) and locally convex space (l.c.s.), characterization of local base at θ in an l.t.s. and in an l.c.s., basic properties, descriptions of finest linear and locally convex topologies, bounded and totally bounded sets.
- Seminorm, Minkowski functional, seminorm characterizations of l.c.s., seminormable spaces. Uniformity and metrizability of l.t.s. and l.c.s.. Completeness, F-space and Frechet space. Examples : $L_p(0,1)$ $(0 < p < 1), C(X)$ (where X is a locally compact, σ-compact and noncompact T_2 space), K^I, $C[a,b]$ with pointwise convergent topology.
- Linear maps and linear functionals, bounded linear maps. Product and Quotient spaces. Finite dimensional l.t.s., Riesz theorem. Banach Steinhans theorem, open mapping and closed graph theorems for F-spaces.

References

[1] John Horvath; Topological Vector Spaces and Distributions; Addison-Wesley Publishing Co. (1966).

[2] J.L.Kelly and Isaac Nomioka; Linear Topological Spaces; D.Van Nostrand Co.Inc. (1963).

[3] Albert Wilansky; Modern Methods in Topological Vector Spaces; McGraw Hill Int. Book Co. (1978).

[4] Charles Swartz; An Introduction to functional Analysis; Marcel Dekker, Inc. (1992).

[5] John B.Conway; A Course in functional Analysis; Springer International Edition (1990).

[6] W.Rudin; Functional Analysis; Tata McGraw-Hill, New Delhi, (1987).

Course Structure | Elective I | Elective II

Advanced Functional Analysis - II

Semester : IV	Subject Code : 202
Course ID : PM4/E2/202	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Linear manifold, affine hyperplane, Geometric form of Hahn-Banach theorem, separation form of Hahn-Banach theorem and some of its consequences including Extension theorem.
- Algebraic dual and topological dual of a locally convex space, weak topology and weak-* topology. Polar, bipolar theorem, Banach-Alaoglu theorem, Alaoglu-Bourbaki theorem. Extreme points and extreme sets, Krein-Milman theorem. Strong topology, Polar topology and Mackey topology, Mackey-Arens theorem, Machey's theorem.
- Bornivorous and barrel subset, Banach-Mackey theorem, bornological and barrelled spaces, infrabarrelled spaces, bidual, semi-reflexivity and reflexivity. Montel spaces and Schwarz spaces. Inverse limit and inductive limit of locally convex spaces. Distribution —— an introduction and certain basic results.

References

[1] John Horvath; Topological Vector Spaces and Distributions; Addison-Wesley Publishing Co. (1966).

[2] J.L.Kelly and Isaac Nomioka; Linear Topological Spaces; D.Van Nostrand Co.Inc. (1963).

[3] Albert Wilansky; Modern Methods in Topological Vector Spaces; McGraw Hill Int. Book Co. (1978).

[4] Charles Swartz; An Introduction to functional Analysis; Marcel Dekker, Inc. (1992).

[5] John B.Conway; A Course in functional Analysis; Springer International Edition (1990).

[6] W.Rudin; Functional Analysis; Tata McGraw-Hill, New Delhi, (1987).

Course Structure | Elective I | Elective II

Fourier Analysis - I

Semester : III Course ID : PM3/E2/203	Subject Code : 203 Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Trigonometric polynomials, Fourier coefficients, Fourier Series, Convolution, Riemann-Lebesgue Lemma, Plancherel identity, Dirichlet kernel, Fejer kernel, Summability and Convergence of Fourier series.
- Convolution, Approximate identity, Fourier transform, Inversion Theorem, Plancherel Theorem, Fourier transform of L^p functions for $1 < p < 2$, Schwartz space.
- Poisson summation formula, Interpolation theorems, Paley-Wiener theorem, Wiener's Tauberian theorem, Spherical harmonics and symmetry properties of Fourier transform.

References

[1] H. Dym and H. P. McKean; Fourier Series and Integrals; Probability and Mathematical Statistics, No. 14, Academic Press, New York-London, 1972.

[2] Yitzhak Katznelson; An Introduction to Harmonic Analysis; Third Edition, Cambridge Mathematical Library, Cambridge University Press, Cambridge, 2004.

[3] Walter Rudin; Functional Analysis; Second edition, International Series in Pure and Applied Mathematics, McGraw-Hill, Inc., New York, 1991.

[4] Elias M. Stein and Rami Shakarchi; Fourier Analysis - An introduction; Princeton Lectures in Analysis, 1, Princeton University press, Princeton, NJ, 2003.

[5] Elias M. Stein & Guido Weiss; Introduction to Fourier Analysis on Euclidean Spaces; Princeton Mathematical Series, No. 32, Princeton University Press, Princeton, NJ, 1971.

Course Structure | Elective I | Elective II

Fourier Analysis - II

Semester : IV	Subject Code : 203
Course ID : PM4/E2/203	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Topological Groups, Haar measure, L^p spaces, Convolution and Approximate Identities.
- Unitary Representations, Equivalence of Representations, Irreducibility, Schur's Lemma, Cyclic Representations.
- Representation Theory of Compact Groups, Schur's Orthogonality Relations, Character of a Representation, Peter-Weyl theorem.
- Linear Lie groups, the Exponential map, Lie algebra of linear Lie group, Calculus on linear Lie group, Invariant differential operators, Finite dimensional representations of a linear Lie group and its Lie algebra.

References

[1] S. C. Bagchi, S. Madan, A. Sitaram and U. B. Tiwari; A first course on representation theory and linear Lie groups; Universities Press, Hyderabad, 2000.

[2] Gerald B. Folland; A Course in Abstract Harmonis Analysis; Second edition, Textbooks in Mathematics, CRC Press, Boca Raton, FL, 2016.

[3] Brian C. Hall; Lie Groups, Lie Algebras and Representations, An Elementary Introduction; second Edition, Graduate Texts in Mathematics, 222, Springer, Cham, 2015.

[4] Mitsuo Sugiura; Unitary Representations and Harmonic Analysis, An Introduction; Second Edition, North-Holland Mathematical Library, 44, north-Holland Publishing Co., Amsterdam, Kodansah, Ltd., Tokyo, 1990.

Course Structure | Elective I | Elective II

Rings of Continuous Functions - I

Semester : III	Subject Code : 204
Course ID : PM3/E2/204	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- $C(X)$ and $C^*(X)$. C-embedded and C^*-embedded sets in X, Urysohn's extension theorem. Pseudo–compact spaces — their characterizations. Adequacy of Tychonoff X for consideration of $C(X)$, $C^*(X)$ – M.H. Stones theorem. Z-filters, Z-ultrafilters on X, their duality with ideals, Z-ideals and maximal ideals of $C(X)$. Structure spaces of $C(X)$, $C^*(X)$, hull-kernel topology. Banach-Stone theorem. Wall man compactification, the partially ordered set $K(X)$ of all compactifications of X, its lattice structure. Gelfand-Kolmogoroff theorem. Variant constructions of βX achieved as,
  (i) The structure space of $C(X)$,
  (ii) The structure space of $C^*(X)$,
  (iii) The space of all nonzero real valued homomorphisms on $C^*(X)$.
  The spaces $\beta\mathbb{N}$, $\beta\mathbb{Q}$ and $\beta\mathbb{R}$.

References

[1] Gillman and Jerison; Rings of continuous functions; Springer-verlag, N.Y. Heidelberg, Berlin, 1976.

[2] Charles E. Aull; Rings of continuous functions; Marcel Dekker. Inc. 1985.

[3] R. C. Walker; The Stone-Čech compactification; Springer, N.Y. 1974.

[4] R. E. Chandler; Hausdörff compactifications; Marcel Dekker, Inc. N.Y. 1976.

[5] J. Dugundji; Topology; Boston, allyn and Bacon, 1966.

[6] Porter and Woods; Extensions and Absolutes of Hausdörff spaces; Springer, 1988.

[7] Franklin Mendivil; Function Algebras & The Lattice of Compactifications; Proceedings of the American Mathematical Society, Vol-127, No. 6, Pages: 1863-1871, 1999.

[8] Gillman and Kohls; Convex and pseudo-prime ideals in rings of continuous functions; Math-zeitschr, 72, 399-409, 1960.

Course Structure | Elective I | Elective II

Rings of Continuous Functions - II

Semester : IV	Subject Code : 204
Course ID : PM4/E2/204	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Quotient rings of $C(X)$, real and hyper real maximal ideals, convex and absolutely convex ideals, Archimedean and non Archimedean quotient fields of $C(X)$. Real compact spaces, restoration of real compact spaces X from $C(X)$, Hewitt's isomorphism theorem, Hewitt real compactification υX of X, properties of real compact spaces. Cardinals of closed sets in βX, nondiscrete closed sets in $\beta X \setminus \upsilon X$, restoration of 1st countable spaces X from $C^*(X)$, unique determination of metric spaces X from $C(X)$. Extremally disconnected spaces, basically disconnected spaces, P-spaces and F-spaces – interrelation between these spaces, conditionally complete and σ-conditionally complete lattice structure of $C(X)$ and $C^*(X)$. The rings $C_k(X)$ and $C_\infty(X)$ —— their interactions will locally compact spaces. Ordinal spaces $\omega_1, \omega_1 + 1, \omega_{\alpha+1}$ —— rings of continuous functions on these spaces. Tychonoff plank.

References

[1] Gillman and Jerison; Rings of continuous functions; Springer-verlag, N.Y. Heidelberg, Berlin, 1976.

[2] Charles E. Aull; Rings of continuous functions; Marcel Dekker. Inc. 1985.

[3] R. C. Walker; The Stone-Čech compactification; Springer, N.Y. 1974.

[4] R. E. Chandler; Hausdörff compactifications; Marcel Dekker, Inc. N.Y. 1976.

[5] J. Dugundji; Topology; Boston, allyn and Bacon, 1966.

[6] Porter and Woods; Extensions and Absolutes of Hausdörff spaces; Springer, 1988.

[7] Franklin Mendivil; Function Algebras & The Lattice of Compactifications; Proceedings of the American Mathematical Society, Vol-127, No. 6, Pages: 1863-1871, 1999.

[8] Gillman and Kohls; Convex and pseudo-prime ideals in rings of continuous functions; Math-zeitschr, 72, 399-409, 1960.

Course Structure | Elective I | Elective II

Structures on Manifolds - I

Semester : III	Subject Code : 205
Course ID : PM3/E2/205	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Review of Riemannian manifolds :** Riemannian manifolds, Affine Connections (Koszul), Torsion and Curvature tensor field on Affine Connection, Covariant Differential.
- **Almost Complex Manifolds :** Introduction, algebraic Preliminaries, Nijenhuis tensor, Eigen values of the complex structure, Existence theorem and Integrability condition of an almost complex structure, Contravariant and covariant almost analytic vector field, Complex manifold.
- **Almost Hermite Manifolds :** Introduction, Nijenhuis tensor, curvature tensor, Holomorphic sectional curvature, Linear connection in an almost Hermite manifold.
- **Kähler Manifolds :** Introduction, Holomorphic sectional curvature, Bochner curvature tensor, Affine connection in Kähler manifolds, Conformally flat Kähler manifolds, Projective correspondence between two Kähler manifolds.
- **Nearly Kähler Manifolds :** Definition, curvature identities.
- **Para Kähler Manifolds :** Introduction, curvature identities, conformal flatness of para Kähler manifolds.
- **Submanifolds of Kähler manifolds :** Käehlerian submanifolds, Anti-invariant submanifolds of Käehlerian manifolds, CR-submanifolds of Käehlenian manifolds.

References

[1] R.S.Mishra; Structures on a Differentiable Manifold and Their Applications; Chandrama Prakashan, Allahabad, 1984.

[2] K.Yano and M.Kon; Structures on Manifolds; World Scientific, 1984.

[3] S.S. Chern; Complex Manifolds Without Potential Theory; New York, Springer-Verlag, 1979.

[4] P. Griffiths and J. Harris; Principles of Algebraic Geometry; New York, John Wiley & Sons, 1978.

[5] R. C. Gunning; Lectures on Riemann Surfaces; Princeton, Princeton University Press, 1966.

[6] S. Kobayashi and K. Nomizu; Foundations of Differential Geometry; New York, Interscience Publishers, 1969.

[7] A. Moroianu; Lectures on Kähler Geometry; www.arxiv.org/math.DG/0402223.

[8] J. Morrow and K. Kodaira; Complex Manifolds; New York, Holt Rinehart and Winston, 1971.

[9] R. O. Wells, Jr.; Differential Analysis on Complex Manifolds; New York, Springer-Verlag, 1980.

[10] F. Zheng; Complex Differential Geometry; Providence, American Mathematical Society, 2000.

Course Structure | Elective I | Elective II

Structures on Manifolds - II

Semester : IV	Subject Code : 205
Course ID : PM4/E2/205	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Contact Manifolds :** Contact manifold, contact metric manifold, almost contact manifold, Torsion tensor of almost contact metric manifold, Killing vector field, properties of φ, the tensor field h, some curvature properties of contact metric manifold.
- **K-contact Manifolds :** Characterizations of K-contact manifolds, some curvature properties of K-contact manifolds, sectional curvature of K-contact manifolds, Locally symmetric and Ricci symmetric K-contact manifolds, semi-symmetric and Ricci - semisymmetric K-contact manifolds.
- **Sasakian manifolds :** Introduction, some curvature properties, φ sectional curvature of a Sasakian manifold, semi-symmetric and Weyl semi-symmetric Sasakian manifolds, C-Bochner curvature tensor, D-Homothetic Deformation.
- **$N(k)$-Contact Metric Manifolds :** k-nullity distribution, η-Einstein $N(k)$-Contact Metric manifolds, Conformally flat $N(k)$-contact metric manifolds, some curvature properties.
- **Para-contact Structure :** Almost para-contact structure, Torsion tensor fields, Examples of paracontact manifolds, P-Sasakian manifolds.
- **Submanifolds of Sasakian Manifolds :** Invariant submanifolds of Sasakian manifolds, Anti-invariant submanifolds tangent to the structure vector field of Sasakian manifolds, Anti-invariant submanifolds normal to the structure vector field of Sasakian manifolds.

References

[1] R.S.Mishra; Structure on a Differentiable manifold and their Applications; Chandrama Prakashani, Allahabad, 1984.

[2] K.Yano and M.Kon; Structures on Manifolds; World Scientific, 1984.

[3] Blair, D. E.; Contact manifolds in Riemannian geometry; Lecture note in Math., 509, Springer-Verlag, Berlin-New York, 1976.

[4] Blair, D. E.; Riemannian geometry of contact and symplectic manifolds; Progress in Math., 203, Birkhauser Boston, Inc., Boston, 2002.

Course Structure | Elective I | Elective II

Advanced Number Theory - I

Semester : III	Subject Code : 206
Course ID : PM3/E2/206	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Arithmetic Functions :** The Mobius function, The Euler totinent function, The Dirichlet product of arithmetical functions, Liouville's functions, generalized convolution.
- **Distribution of Prime Numbers :** Chebyshev's functions, Some equivalent forms of the prime number theorem, Shapiro's Tauberian theorem.
- Dirichlet's theorem on primes in arithmetic progressions.
- Dirichlet series and Euler products, Zeta functions, Prime number theorem, Riemann Hypothesis concerning zeros of zeta function.

References

[1] T. M. Apostol; Introduction to Analytic Number Theory; Narosa Publishing House, Springer International Student Edition.

[2] G. H. Hardy & E. M. Wright; An Introduction to the Theory of Numbers; 4th edition, Oxford : Clarendon Press (1960).

[3] J. P. Serre; A Course in Arithmetic; Narosa Publishing House (1973).

[4] Donald J. Newman; Analytic Number Theory; Springer (1998).

Course Structure | Elective I | Elective II

Advanced Number Theory - II

Semester : IV	Subject Code : 206
Course ID : PM4/E2/206	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **The development of Algebraic Number theory :** Algebraic Integers, Quadratic Fields, Quadratic integers, Geometric representation, Factorization in quadratic fields, non-unique factorization and ideals.
- **Ideals in quadratic fields :** Arithmetic of ideals, Lattice and Ideals, unique factorization of ideals, application of unique factorization, divisibility of Diophantine equations, factorization of rational primes, class structure and class numbers, finiteness of class numbers, norm of an ideal, bases and discriminants, the correspondence between forms and fields.
- **Geometry of Numbers :** The motivation of the problem, quadratic forms, Minkowski's fundamental theorem, Minkowski's theorem for lattices, sum of two and four squares, linear forms, sum and product of linear forms, Dirichlet's theorem, LLL-reduced base, LLL algorithm.
- **p-adic numbers and valuations :** History, the p-adic numbers, an informal introduction, the formal development, convergence, congruences and p-adic numbers, Hasse's principle, Hasses-Minkowski Theorem, Valuation and Algebraic Number theory.
- **Algorithmic Number Theory :** Lagendre-Jacobi-Kronecker Symbols, Shanks-Tonelli Algorithm, Solving Polynomial equations modulo p, Some primality testing algorithms, Some factorization algorithms.
- **Application to Diophantine Equations :** Lucas-Lehmer Theory, Generalized Ramanujan-Nagell Equations, Bachet's equation, The fermat equation, catalan and ABC Conjecture.
- **Elliptic curves :** The basics, Mazur, Siegel and Reduction, Applications: Factoring & Primality testing, Elliptic Curve Cryptography.

References

[1] Kenneth Ireland & Michael Rosen; A Classical Introduction to Modern Number Theory; 2nd edition, Springer-verlag.

[2] Richard A Mollin; Advanced Number Theory with Applications; CRC Press, A Chapman & Hall Book.

[3] Saban Alaca, Kenneth S Williams; Introduction to Algebraic Number Theory; Cambridge University Press.

[4] Jay R Goldman; The Queen of Mathematics : a historically motivated guide to number theory; A K Peters Ltd.

[5] Henri Cohere; A course in Computational Number Theory; Springer Verlag, 1996.

[6] Jurgen Neukirch; Algebraic Number Theory; Springer Verlag, 1999.

[7] Henri Cohur; Number Theory. Vol-1, Tools of Diaphantine equations; Graduate Text in Mathematics, Springer Verlag, 2007.

Course Structure | Elective I | Elective II

Advanced General Topology - I

Semester : III	Subject Code : 207
Course ID : PM3/E2/207	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Uniformity :** Uniformity and its uniform topology, neighbourhoods, bases and subbases, uniform continuity, product uniformities, uniform isomorphism, relativization and products.

  Characterization of metrizability, uniformity of pseudometric spaces, uniformity generated by a family of pseudometrics, the gauge of uniformity. Completeness, Cauchy net, Cauchy filter, complete spaces, extension of mappings, completion-existence and uniqueness.

- **Compactness and uniformity :** diagonal uniformities, uniformity via uniform covers.

- **Proximity Spaces :** Topology induced by a proximity, subspaces and products of proximity spaces, elementary proximity, p-continuity and p-isomorphism, Compactification of proximity spaces-clusters and ultrafilters, Smirnov's theorem.

- **Ordinal Numbers and Ordinal Spaces :** Definition and properties of ordinal numbers. Cardinal numbers vis-à-vis ordinal numbers, Ordinal spaces, topological properties of ordinal spaces — ω_1 and ω_2 (in particular).

References

[1] J. Dugundji; Topology; Prentice-Hall of India Pvt. Ltd. (1975).

[2] R. Engelking; Outline of General Topology; North-Holland Publishing Co., Amsterdam (1968).

[3] Ioan James; Topologies and Uniformities; Springer-Verlag (1999).

[4] J. L. Kelley; General Topology; D.Van Nostrand Co. Inc. (1955).

[5] Jun Iti Nagata; Modern General Topology; North-Holland Pub. Amsterdam (1985).

[6] S. A. Naimpally & B. D. Warrack; Proximity Spaces; Cambridge University Press (1970).

[7] S. Willard; General Topology; Addison-Wesley Publishing Co. (1970).

Course Structure | Elective I | Elective II

Advanced General Topology - II

Semester : IV	Subject Code : 207
Course ID : PM4/E2/207	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- **Paracompactness :** Types of refinements, paracompactness in terms of open locally finite refinements, Michael's theorem, fully normal spaces, Stone's coincidence theorem, paracompactness in terms of open delta refinements, cushioned refinements etc. A. H. Stone's theorem —— every metric space is paracompact, partition of unity, properties of paracompact spaces with regard to subspaces, product etc.

- **Function Spaces :** Pointwise convergence topology and uniformity, compact-open topology, uniqueness of jointly continuous topology, uniform convergence on a family of sets, completeness, uniform convergence on compacta, K-spaces, compactness and equicontinuity. The Ascoli theorem, Even continuity, topological Ascoli theorem, basis for Z^Y, compact subsets of Z^Y, sequential convergence in the c-topology, metric topologies —— relation to the c-topology, pointwise convergence, comparison of topologies in Z^Y. The spaces $C(Y)$ — continuity of the algebraic operations, algebras in $\widehat{C}(Y, C)$, Stone-Weierstrass theorem, the metric space $C(Y)$, embedding of Y in $C(Y)$, The ring $\widehat{C}(Y)$.

- **Metrization :** Metrization theorems of Nagata–Smirnov, Bing, Smirnov, A.H.Stone, Arhangeliskii etc.

- **Elements of Dimension Theory :** Menger-Urysohn dimension (the small inductive dimension) of a space, $indX$ and $IndX$, $\dim X$, associated results, specially in connection with 0-dimensional or totally disconnected spaces and βX etc.

References

[1] J. Dugundji; Topology; Prentice-Hall of India Pvt. Ltd. (1975).

[2] R. Engelking; Outline of General Topology; North-Holland Publishing Co, Amsterdam. (1968).

[3] J. L. Kelley; General Topology; D.Van Nostrand Co. Inc. (1955).

[4] Jun Iti Nagata; Modern General Topology; North-Holland Pub. Amsterdam (1985).

[5] S.Willard; General Topology; Addison-Wesley Publishing Co. (1970).

[6] W. Hurewicz and H. Wallman; Dimension Theory; Princeton University Press (1948).

Course Structure | Elective I | Elective II

Theory of Linear Operators - I

Semester : III	Subject Code : 208
Course ID : PM3/E2/208	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Spectral theorem in normed linear spaces, resolvent set and spectrum. Spectral properties of bounded linear operators. Properties of resolvent and spectrum. Spectral mapping theorem for polynomials. Spectral radius of a bounded linear operator on a complex Banach space. Certain concepts of the theory of Banach Algebras.

- General properties of compact like operators. Spectral properties of compact linear operators on normed spaces. Behaviours of compact linear operators with respect to solvability of operator equations.

- Spectral properties of bounded self-adjoint operators on a complex Hilbert space. Positive operators. Monotone sequence theorem for bounded self-adjoint operators on a complex Hilbert space. Square root of a positive operator. Projection operators. Spectral family of a bounded self-adjoint linear operator and its properties. Spectral representation of bounded self adjoint linear operators. Spectral theorem.

References

[1] E.Kreyszig; Introductory Functional Analysis with applications; John-Wiley and sons, N.Y. (1978)

[2] P.R.Halmos; Introduction to Hilbert spaces and the theory of Spectral Multiplicity; Cheilsea Publishing co., N.Y. (1957).

[3] P.R.Halmos; A Hilbert space Problem Book; D.Van Nostrand Co. Inc.(1967).

[4] N.Dunford and J.T.Schwartz; Linear Operators, 3 Vols.; Interscience Wiley, N.Y.(1958).

[5] G.Bachman and L.Narici; Functional Analysis; Academic Press, N.Y.(1966).

[6] N.I.Akhiezer and M.Glazman; Theory of Linear Operators in Hilbert Spaces; Fredelick Ungar Pub. Co. Vol-I (1961), Vol-II (1963).

Course Structure | Elective I | Elective II

Theory of Linear Operators - II

Semester : IV	Subject Code : 208
Course ID : PM4/E2/208	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Spectral measures. Spectral Integrals. Description of the spectral subspaces. Characterizations of the spectral subspaces. The spectral theorem for bounded normal operators. Unbounded linear operators in Hilbert spaces. Hellinger-Toeplitz theorem. Hilbert adjoint operator. Symmetric and self-adjoint linear operators. Inverse of the Hilbert adjoint operator. Closed linear operators and closures. Hilbert adjoint of the closures. Spectrum of an unbounded self-adjoint operator. Spectral theorems for unitary and self-adjoint linear operators. Multiplication and differentiation operators.

References

[1] E.Kreyszig; Introductory Functional Analysis with applications; John-Wiley and sons, N.Y. (1978)

[2] P.R.Halmos; Introduction to Hilbert spaces and the theory of Spectral Multiplicity; Cheilsea Publishing co., N.Y. (1957).

[3] P.R.Halmos; A Hilbert space Problem Book; D.Van Nostrand Co. Inc.(1967).

[4] N.Dunford and J.T.Schwartz; Linear Operators, 3 Vols.; Interscience Wiley, N.Y.(1958).

[5] G.Bachman and L.Narici; Functional Analysis; Academic Press, N.Y.(1966).

[6] N.I.Akhiezer and M.Glazman; Theory of Linear Operators in Hilbert Spaces; Fredelick Ungar Pub. Co. Vol-I (1961), Vol-II (1963).

Course Structure | Elective I | Elective II

Banach Algebra - I

Semester : III	Subject Code : 209
Course ID : PM3/E2/209	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- General preliminaries on Banach Algebras. Definitions and some examples. Regular and singular elements. Topological divisors of zero. The Spectrum. The formula for the Spectral radius.
- The radical, semi-simplicity, ideals, maximal ideals space, structure of semisimple Banach algebras.
- The carrier space and the Gelfand representation theorem, algebras of functions, The Silov boundary, representation of the carrier space, homomorphisms of certain function algebras into a Banach algebra, direct-sum decomposition and related results.
- Involution in Banach algebras, the Gelfand-Neumark theorem.

References

[1] C.E.Rickart; General theory of Banach algebras; D.Van Nostrand Company, INC.

[2] G.F.Simmons; Topology and Modern Analysis, McGraw-Hill book company (1963)

[3] S.Sakai; C^*-Algebras & W^*-Algebras ; Springer-Verlag, 1971.

Course Structure | Elective I | Elective II

Banach Algebra - II

Semester : IV Course ID : PM4/E2/209	Subject Code : 209 Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Commutative-*-algebras, Self-dual vector spaces and *-representations, positive functionals and *-representations on Hilbert space, General properties of B^*-algebras, structure of ideals and representations of B^*-algebras.
- **Algebras of operators :** Elements of algebras of compact operators, C^*-algebra, W^*-algebra, positive elements and positive linear functionals on C^*-algebra, weak topology and various topologies on W^*-algebra, ideals in W^*-algebra, spectral resolution of self-adjoint elements in a W^*-algebra.

References

[1] C.E.Rickart; General theory of Banach algebras; D.Van Nostrand Company, INC.

[2] G.F.Simmons; Topology and Modern Analysis, McGraw-Hill book company (1963)

[3] S.Sakai; C^*-Algebras & W^*-Algebras ; Springer-Verlag, 1971.

Course Structure | Elective I | Elective II

Non-standard Analysis - I

Semester : III	Subject Code : 210
Course ID : PM3/E2/210	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Real, hyper-real, standard, nonstandard numbers. Infinitesimals and infinitely large numbers- method of their actual constructions. The system $^*\mathbb{R}$ of hyper-realnumbers. Monads of points of $^*\mathbb{R}$. The relational system associated with $\mathbb{R}$ and $^*\mathbb{R}$; star transforms of relations and functions in $\mathbb{R}$. Statement and proof of Heine-Borel theorem. Convergence of sequences and their subsequences - a nonstandard approch. Non standard treatment of limit, continuity and differentiability of a function in $\mathbb{R}$ - nonstandard proofs of intermediate value theorem, extreme value theorem, chain rule.

References

[1] A.E. Hurd and P.A. Hurd and P.A. Loeb; An introduction to Non-standard Real Analysis; Academic Press, 1985.

[2] H. J. Keisler-Prindle, Weber and Schmidst Boston; Foundation of infinitesimal Calculus; 1976.

[3] A. Robinson; Nonstandard Analysis; North Holland, Amsterdam, 1966.

[4] K. Stroyan and W. A. J. Luxemberg; Introduction to the theory of infinitesimals; Academic Press, New York, 1976.

[5] M. Davis; Applied Non-standard Analysis; Wiley, New York, 1977.

[6] A. Robinson and A. H. Light Stone; Non-standard fields and Asymptotic expansion; North Holland, Amsterdam, 1975.

Course Structure | Elective I | Elective II

Non-standard Analysis - II

Semester : IV	Subject Code : 210
Course ID : PM4/E2/210	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Non-standard description of Riemann integrals of continuous functions over closed bounded intervals, Keisler's infinite sum theorem, fundamental theorem of integral calculus - a non-standard proof. Non standard approach to pointwise and uniform convergence of sequences of functions, proof of Dini's theorem and Arzela-Ascoli theorem. Robinson's non-standard proof of Cauchy-Peano existence theorem for ordinary differential equation. Non-standard descriptions of differentiability and uniform differentiability of function of two variables. Implicit function theorem for two variables - a non-standard proof. Equality of the mixed partial derivatives - a non-standard proof. Hyperreal characterisation of line and double integral. Statement and proof of Green's theorem by non-standard method.

References

[1] A.E. Hurd and P.A. Hurd and P.A. Loeb; An introduction to Non-standard Real Analysis; Academic Press, 1985.

[2] H. J. Keisler-Prindle, Weber and Schmidst Boston; Foundation of infinitesimal Calculus; 1976.

[3] A. Robinson; Nonstandard Analysis; North Holland, Amsterdam, 1966.

[4] K. Stroyan and W. A. J. Luxemberg; Introduction to the theory of infinitesimals; Academic Press, New York, 1976.

[5] M. Davis; Applied Non-standard Analysis; Wiley, New York, 1977.

[6] A. Robinson and A. H. Light Stone; Non-standard fields and Asymptotic expansion; North Holland, Amsterdam, 1975.

Course Structure | Elective I | Elective II

Theory of Frames - I

Semester : III	Subject Code : 211
Course ID : PM3/E2/211	Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Topological spaces and lattices, Frames and frame maps, Prime elements and spectrum of a frame, Regular, completely regular, normal frames, Compact frames and compactification of a frame, Continuous frames, Uniform and nearness frames, Paracompact frames.

References

[1] Picado, J., Pultr, A.; Frames and Locales : Topology without points; Frontiers in Mathematics, Springer, Basel (2012).

[2] Johnstone, P.T.; Stone Spaces; Cambridge Univ. Press, Cambridge (1982).

[3] Banaschewski, B.; The Real Numbers in Pointfree Topology, In: Departmento de Matemática da Univeridade de Coimbra, Textos de Matemática, Série B, no. **12**, 94pp. (1997).

[4] Ball, R.N., Walters-Wayland, J.; C- and C^*- quotients in pointfree topology. Dissertationes Math. (Rozprawy Mat.), Warszawa, vol. **412**, 62pp. (2002)

Course Structure | Elective I | Elective II

Theory of Frames - II

Semester : IV Course ID : PM4/E2/211	Subject Code : 211 Full Marks : 50
Minimum number of classes required : 70	

Course Structure | Elective I | Elective II

- Generating set of a frame, Congruence of a frame, Quotient and Nucleus of a frame, Frame of reals and Rings of continuous functions on a frame, C-quotient and C^*-quotient of a frame, Complete separation, Pointfree Uryshon's extension theorem, Uryshon's lemma, Tietze's extension theorem, P-frames, almost P-frames, basically disconnected frames, F-frames etc., Stone-Čech compactification and Hewitt realcompactification of frames.

References

[1] Picado, J., Pultr, A.; Frames and Locales: Topology without points; Frontiers in Mathematics, Springer, Basel (2012).

[2] Johnstone, P.T.; Stone Spaces; Cambridge Univ. Press, Cambridge (1982).

[3] Banaschewski, B.; The Real Numbers in Pointfree Topology, In: Departmento de Matemática da Univeridade de Coimbra, Textos de Matemática, Série B, no. **12**, 94pp. (1997)

[4] Ball, R.N., Walters-Wayland, J.; C- and C^*- quotients in pointfree topology. Dissertationes Math. (Rozprawy Mat.), Warszawa, vol. **412**, 62pp. (2002)

Course Structure | Elective I | Elective II

University of Calcutta

Syllabus for Choice Based Credit Course (CBCC) on

"*Rudiments of Pure Mathematics*"

offered by Department of Pure Mathematics

Under

CBCS System

2018

Algebra, Discrete Mathematics & Analysis

Semester : III	Credits : 4
Code : CBCC	Full Marks : 50
Minimum number of classes required : 60	

Unit-1 : Abstract Algebra [10 Marks]

- Group theory, properties of group, cyclic group, quotient group, group homomorphism and isomorphism, Sylow's theorems.
- Ring theory, properties of ring, integral domain, ideal, quotient ring, ring homomorphism, polynomial ring, field, few applications of group, ring, field.

Unit-2 : Graph Theory [10 Marks]

- Definition of undirected graphs, Using of graphs to solve different puzzles and problems. Multigraphs. Walks, Trails, Paths, Circuits and cycles, Eulerian circuits and paths. Eulerian graphs, example of Eulerian graphs. Hamiltonian cycles and Hamiltonian graphs.
- Weighted graphs and Travelling salespersons Problem. Dijkstr's algorithm to find shortest path.
- Definition of Trees and their elementary properties. Definition of Planar graphs, Kuratowski's graphs.

Unit-3 : Combinatorics [10 Marks]

- Revisited : Permutations and combinations; Binomial coefficients and Pascal's Triangle. Basic counting principle, The Pigeonhole Principle and its applications.

Unit-4 : Calculus (functions from $\mathbb{R}^n$ to $\mathbb{R}$) [10 Marks]

- Real valued functions of several real variables (i.e., $f : \mathbb{R}^n \to \mathbb{R}$) : scalar field; limit and continuity of scalar fields; derivative of a scalar field with respect to a vector, directional derivatives and partial derivatives, partial derivatives of higher order, total derivatives, gradient of a scalar field, a sufficient condition for differentiability, chain rule for derivatives of a scalar field; Applications to geometry – level sets, tangent planes, etc.

Unit-5 : Metric Space (with special emphasis on $\mathbb{R}$) [10 Marks]

- Metric space and its examples (other than $\mathbb{R}$); subspaces of a metric space; Convergent sequence, Cauchy sequence, Completeness, Cantor's intersection theorem, $\mathbb{R}$ is a complete metric space. Continuous functions on a metric space, sequential criterion of continuous functions (Heine's continuity criterion), Uniform continuity; Compactness and Connectedness of metric spaces, Heine-Borel theorem in $\mathbb{R}$, connected subsets of $\mathbb{R}$. Contraction mappings, Banach fixed point theorem and its applications.

References

[1] D. S. Malik, J. M. Mordeson and M. K. Sen; Fundamentals of Abstract Algebra; McGraw-Hill, 1997.

[2] J. A. Gallian; Contemporary Abstract Algebra (8th Edition), Cengage Learning, 2013.

[3] M. R. Adhikari, A. Adhikari; Basic Modern Algebra with Applications, Springer, 2014.

[4] M. K. Sen, S. Ghosh, P. Mukhopadhyay; Topics in abstract Algebra (2nd edition), University Press, Hyderabad, 2006.

[5] Apostol, T., Calculus (Volume II), John Wiley (student Edition 2), 2007.

[6] Horst, R. Beyer, Calculus and Analysis, Wiley, 2010.

[7] Simmons, G. F., Introduction to Topology and Modern Analysis, McGraw-Hill, 2004.

[8] Kumaresan, S., Topology of Metric Spaces, 2nd Ed., Narosa Publishing House, 2011.

[9] N, Deo; Graph Theory with application to Engineering and Computer Science, Prentice Hall of India, New Delhi, 1990.

[10] John Clark, Derek Allan Holton; A First look at Graph Theory, Allied Publishers Ltd. 1995.

[11] D.S. Malik and M.K. Sen: Discrete Mathematical structure: theory and applications, Thomson, Australia, 2004.

[12] Discrete Mathematics , Schaum's Outline, Tata McGraw-Hill, 2003.