UNIVERSITY OF CALCUTTA

NotificationNo.CSR/05/2022

It is notified for information of all concerned that the Syndicate in its meeting held on 08.12.2021 (vide Item No.46), approved & confirmed the new revised syllabus for **"Bio-Mathematics (MTMA-DSE-A-5-1-TH/TU) and " Boolean Algebra & Automata Theory (MTMA-DSE-B-5-1-TH/TU)"** , both taught at Semester-5 of Mathematics-Honours. (vide CSR/12/2018, dt. 04.6.2018) Courses of Studies under CBCS, in this University, as laid down in the accompanying pamphlet.

The above will take effect from Odd-Semester examination, 2022.

22/2/22

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Registrar

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The 22nd February, 2022

Bio-Mathematics syllabus

Unit-1

[25 classes]

Mathematical biology and the modeling process: an overview.

Simple single-species continuous population models: Malthus model - deduction, analytic solution, behavior of population size as $t \to \infty$, limitations.

Logistic model: formulation, analytic solution, behavior of population size as $t \to \infty$ for different initial population size, carrying capacity.

Gompertz model: deduction, analytic solution.

Qualitative analysis of continuous models: Autonomous first order differential equations - steady states (or fixed points or stationary points or equilibrium points). Biological interpretation of steady states. Stability and asymptotic stability of a steady state. Geometrical analysis. Linearization and stability analysis.

Stability analysis of the steady states of the Malthus model, logistic model and other single-species models.

Allee effect: Basic idea and discussion with the model $\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right)\left(\frac{x}{K_0} - 1\right)$, stability analysis of steady states.

Harvesting problems of a single natural population: Constant-yield harvesting, constant-effort harvesting.

Non-dimensionalisation and reparametrisation in a model: Necessity and application.

Bifurcation: Saddle-node, transcritical and pitchfork bifurcations in one-dimensional case.

Insect outbreak model: The spruce budworm model – deduction, analysis of steady states, presence of saddle-node bifurcation.

Unit-2

[30 classes]

Interacting populations: Predator-prey systems and Lotka-Volterra model – deduction, analytic solution.

Qualitative analysis of continuous models: Two-dimensional nonlinear systems - steady states, nullclines. Linearisation and stability analysis of a steady state, Routh-Hurwitz criteria.

Steady states and linear stability analysis of biologically meaningful steady states of Lotka-Volterra predator-prey model.

Different types of functional responses - Holling type –I, II, III growths.

Improved predator-prey model introducing the logistic growth term for the prey and other predator-prey models, their steady states and linear stability analysis.

Periodic solutions and limit cycles: Statement of Poincaré-Bendixson theorem, Bendixson's negative criterion, Dulac criterion, Hopf bifurcation and Hopf bifurcation theorem (statement only). Examples in the context of biological scenario.

The chemostat: Bacterial growth in a chemostat. Michaelis-Menten kinetics. Formulation of model and steady states.

Populations in competition: Lotka-Volterra competition model, steady states and linear stability analysis.

Epidemic models: Basic terminologies.

SI, Kermack-McKendrick SIR, SIRS models: Formulation of the models. Basic reproduction number.

Unit-3 **[15 classes]**

Discrete single-species models: Linear models, growth models, decay models, discrete logistic model.

Overview of nonlinear difference equations: Steady states and linear stability analysis. Graphical solution of difference equations - cobwebbing.

Systems of two difference equations: Concepts of steady states and linear stability analysis.

Discrete predator-prey model: Model formulation.

Host-parasitoid systems: Nicholson-Bailey model- formulation.

Stability of discrete nonlinear systems: Linear stability analysis of biologically meaningful steady states of discrete predator-prey model and Nicholson-Bailey model.

Reference:

[1] L.E. Keshet, Mathematical Models in Biology, SIAM, 1988.

[2] J. D. Murray, Mathematical Biology, Springer, 1993.

[3] Y.C. Fung, Biomechanics, Springer-Verlag, 1990.

[4] F. Brauer, P.V.D. Driessche and J. Wu, Mathematical Epidemiology, Springer, 2008.

[5] M. Kot, Elements of Mathematical Ecology, Cambridge University Press, 2001.

[6] F. Brauer and C. Castillo-Chavez, Mathematical Models in Population Biology and Epidemiology, Springer, 2012.

[7] S. H. Strogatz, Nonlinear Dynamics and Chaos, Perseus Books, 1994.

[8] N. F. Britton, Essential Mathematical Biology, Springer, 2003.

DSE-B-1 BOOLEAN ALGEBRA AND AUTOMATA THEORY MTMA SEM V

UNIT 1

- Mathematical preliminaries: Set theory (Russell's paradox and a brief introduction to the axiomatic approach toset theory), Relations (including transitive and reflexive-transitive closure needed for derivations), Mapping, Basics of Graph Theory (up to Tree and Rooted Tree, needed for drawing transition diagram and parse trees).

UNIT 2

- Definition, examples and basic properties of ordered sets, maps between ordered sets, duality principle, lattices as ordered sets, lattices as algebraic structures, sublattices, products and homomorphisms, definition, examples and properties of modular and distributive lattices, a brief introduction to Hasse diagram.

UNIT 3

- Boolean algebras, Boolean polynomials and their minimal and maximal forms, Karnaugh diagrams, relation between lattice and Boolean algebra, Logic gates, switching circuits and their applications.

UNIT 4

- Alphabet, strings and their properties (union, concatenation etc.), languages and different types of grammars (a short introduction to Chomsky Hierarchy).
- Finite State Machine (transition table, transition diagram, input/output strings-definitions and examples).
- Regular language and Regular Expression, closure properties of Regular language (emphasis should be given to different examples), Pumping Lemma (Statement and its applications).
- Finite Automata: Difference between FSM and FA, Deterministic and non-deterministic finite automata and their relations with regular language and regular expression, acceptance/rejection of different strings by finite automata, conversion of DFA to NDFA (examples).

UNIT 5

- Context Free Grammar (CFG) and Pushdown Automata (PDA): CFG, parse trees and their applications in detecting ambiguities in CFG, PDA and languages accepted by PDA, Deterministic and Non-deterministic PDA, properties of Context Free Language (CFL), Chomsky Normal Form and Conversion of CFG to CNF, Properties of CFL and closure properties, Pumping Lemma for CFL (statement and its applications).

UNIT 6

- A short introduction to Philosophy of Mathematics and historical background of Turing Machines (TM) (students should learn the role of mathematicians in constructing an Abstract Machine as an Algebraic Structure, which came earlier than the computers used today).

- Introduction to concept of Algorithm in the context of attempts to solve Hilbert's tenth problem (proposed in ICM, 1900).
- TM as a model of computation, variants of TM and their equivalence.
- Recursive and Recursively Enumerable Languages.
  Undecidability, Solvability and Computability: Undecidable problems about Turing Machine (TM) and CFGs, Post Correspondence Problem and application of PCP through examples, Halting Problem.

PROPOSED MARKS DISTRIBUTION FOR THEORY (Full marks – 65)

- MCQ: This section should cover the whole syllabus(20).
- 1 question is to be answered out of 2 questions from UNIT 1,1 question out of 2 from UNIT 2 and 1 question out of 2 from UNIT 3, each question should carry 4 marks.
- 2 questions out of 3 from UNIT 4, each carrying 7 marks ;
- 2 questions out of 3 from UNIT 5, each carrying 6marks;
- 1 question out of 2 from UNIT 6, each carrying 7 marks ;
- Total marks from UNIT 3, 4,5,6 should be 33.